(A sampling of) searches for (non-SUSY) BSM physics with ATLAS

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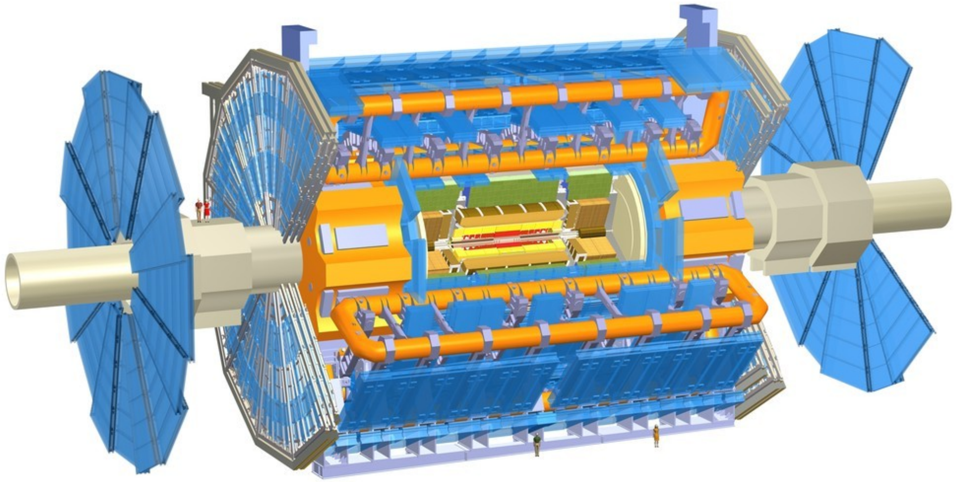
on behalf of ATLAS Collaboration

Higgs and BSM Physics at the LHC
June 24 – 28, ICTP, Trieste

The Abdus Salam International Centre for Theoretical Physics

Overview

"Standard Model" is not so "standard"



Extended scalar sector (2HDM etc)
- ◇ Neutral scalar & pseudo-scalar particles
- ◇ Charged scalar particles
- ◇ Doubly charged scalar particles

Hidden sectors
- ◇ Long-lived particles

Heavy resonances

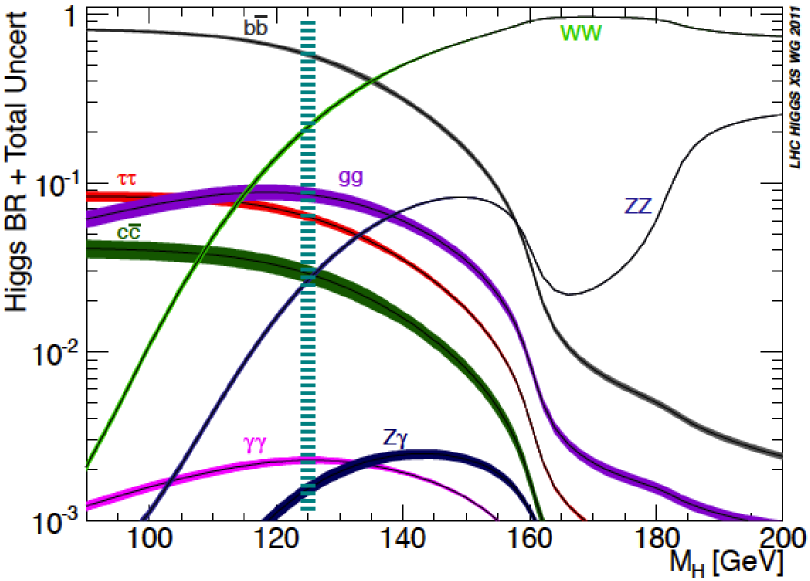
Disclaimer: get prepared for a very biased talk; there are so many results from ATLAS and a selection based on my personal interests and research is presented here

The highlight of 2012

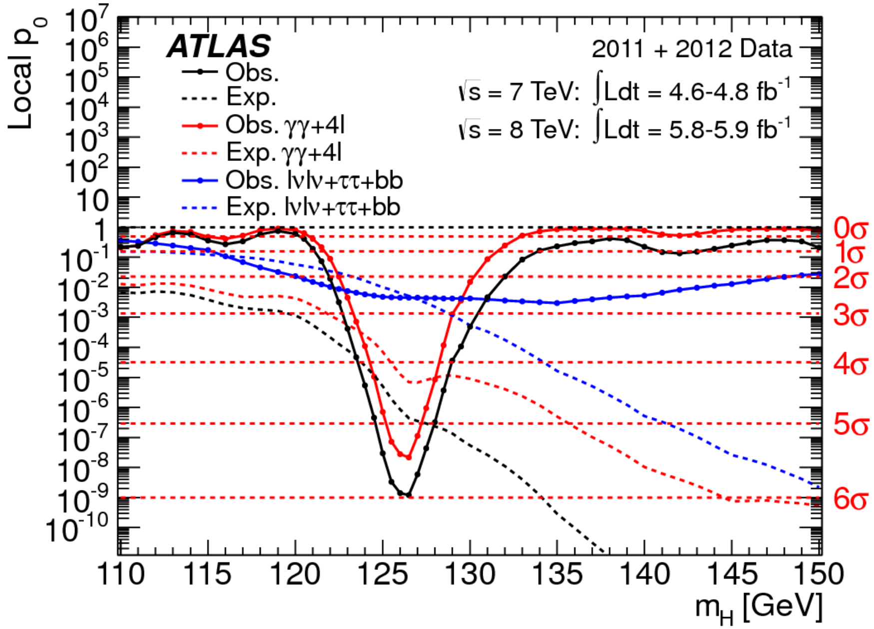
- The discovery of a particle compatible with the SM Higgs boson has been the most important highlight in the field for 2012

◇ Its low mass (~125 GeV) allows its study in many different channels

◇ A large "industry" has been initiated to measure couplings and other properties



Phys. Lett. B 716 (2012) 1-29



A SM Higgs boson?

- Reminder: the majority of experts in this field agree that:

The existence of the SM Higgs boson, i.e. an $SU(2)_L \times U(1)_Y$ (**2**,**1**), is a rather exotic option, which is **most probably not realized in nature**

Fundamental scalars are unstable when considering radiative corrections (**naturalness**)

No fundamental scalars?
(Technicolor, composite Higgs, ...)

New symmetry does the job?
(SUSY, Little Higgs, ...)

New physics, beyond the SM, is implied in all cases

A SM Higgs boson?

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Fundamental scalars are unstable when considering radiative corrections (**naturalness**)

Today we will go the natural way!
For the exotic way on the right, see the talks on Thursday!

Extended Higgs Sectors

- No severe symmetry constraints in the Higgs sector
  - Only $\rho = m_W / (m_Z \cos\theta_W) \approx 1$; this can be satisfied by simply adding singlets or doublets
- The 2 Higgs Doublet Model (2HDM) is one of the simplest options
  - It appears in many BSM scenarios, e.g. SUSY
  - Contains 2 scalars h,H; 1 pseudo-scalar A; 2 charged scalars $H^{\pm}$
  - We can identify h with the newly discovered boson
  - Very interesting to benchmark and motivate searches, but note that if a second doublet exists will probably come with company

For an extensive review of 2HDM pheno see Branco et al, Phys. Rep.,516, p. 1 (2012)

2 Higgs Doublet Models

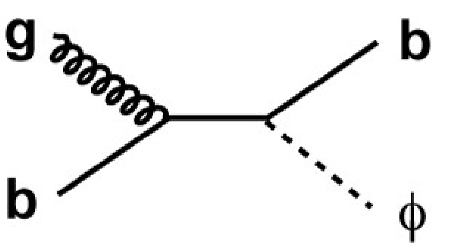
- CP-conservation is usually assumed. In general 2HDM contain FNCN; a Z_2 symmetry is imposed to get rid of them:
  $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$
  - This leaves distinct options to which doublet couples to which particles, and here come the definitions of 2HDM type-I, II etc

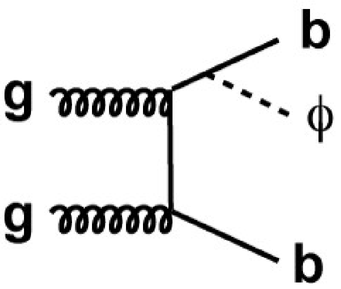
Model	u_R^i	d_R^i	e_R^i
Type I	Φ_2	Φ_2	Φ_2
Type II	Φ_2	Φ_1	Φ_1
Lepton-specific	Φ_2	Φ_2	Φ_1
Flipped	Φ_2	Φ_1	Φ_2

- Under certain assumptions free parameters are the masses of the Higgs bosons and a mixing angle α and the ratio of the two vevs $\tan\beta = u_2/u_1$

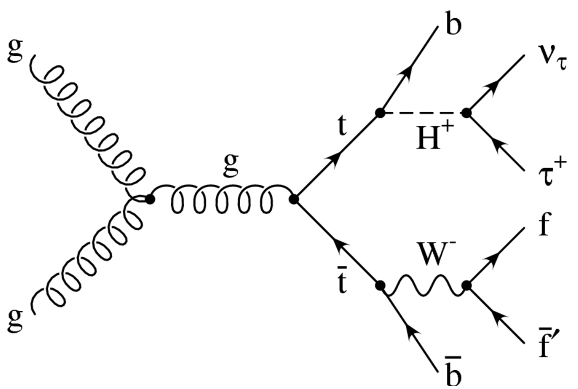
2 Higgs Doublet Models

- Higgs production depends on the Higgs masses, α, tan β and the 2HDM type
- Neutral Higgses:
  - On top of the usual SM Higgs production mechanisms (gluon-fusion, VBF, VH) b-associated production is also relevant here for some parts of the parameter space



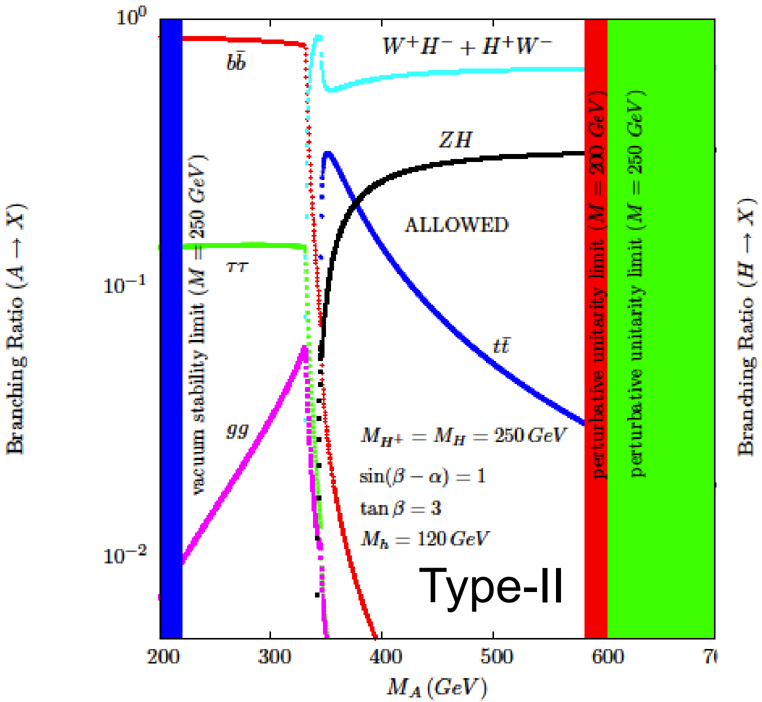


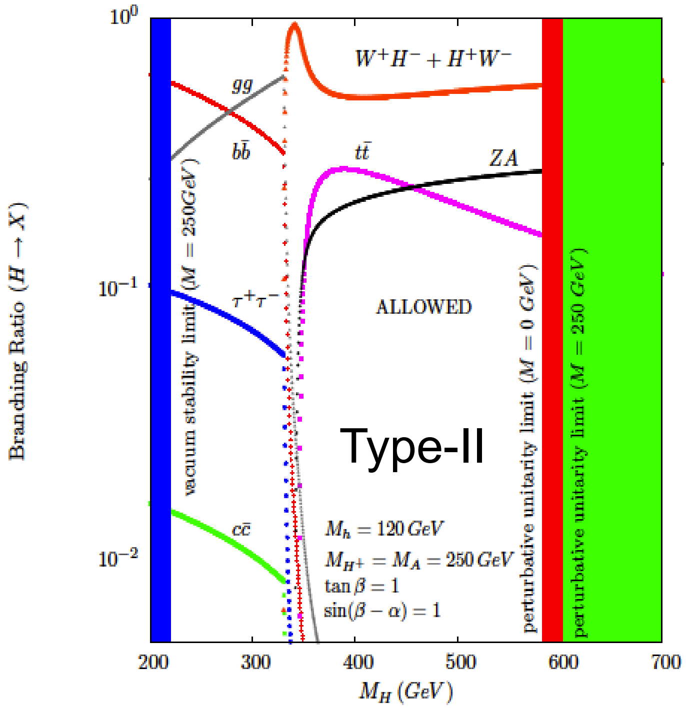
- Charged scalars, if they are light can be produced in top decays

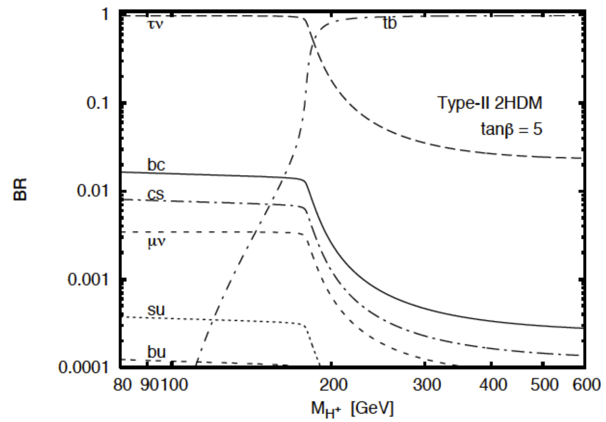


2 Higgs Doublet Models

- Many different options of decays, depending on the model parameters







arXiv:1002.4916

Phys.Rev.D79:055017,2009, Phys. Rep.,516, p. 1 (2012)

h/H/A → τ τ / μ μ

- The heaviest leptons are τs, hence h/H/A → ττ may be important
- H → μμ still interesting: very good mass resolution

H → τ τ		Comment	ATLAS search
τ τ → τ(e/μ) τ(had)	BR ~ 46 %	Most sensitive	✓
τ τ → τ(had) τ(had)	BR ~ 42 %	Important at high mass	✓
τ τ → τ(e) τ(μ)	BR ~ 6 %	Important at low mass	✓
τ τ → τ(μ) τ(μ)/τ(e) τ(e)	BR ~ 6 %		
H → μ μ			✓

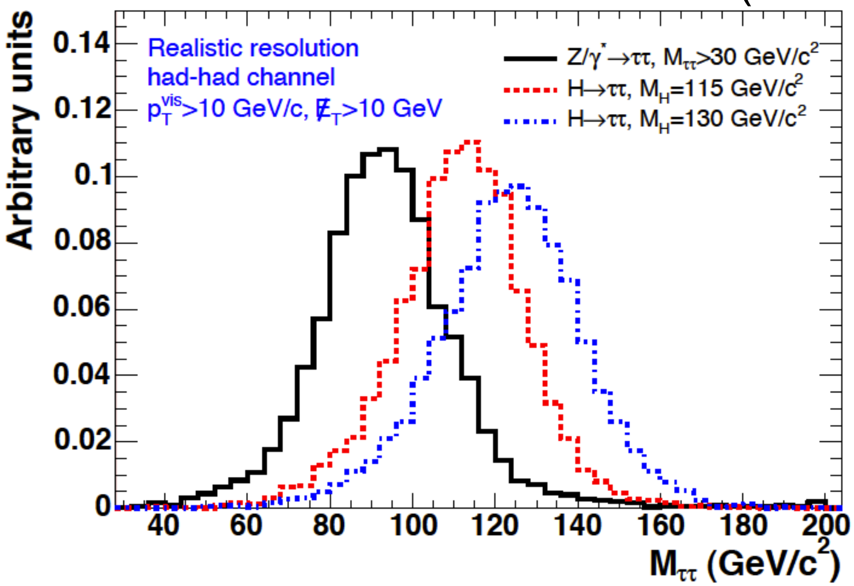
- Focus on 2 production mechanisms: gluon-fusion and in association with b-quarks

Tau Interlude

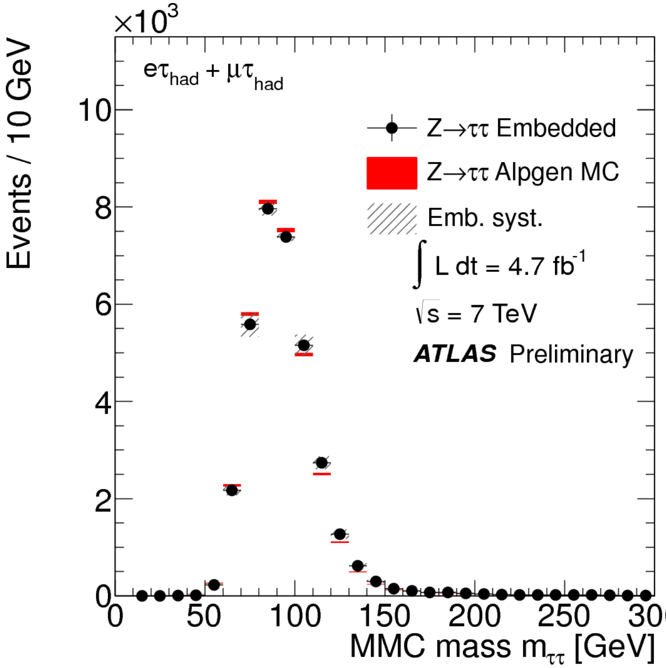
- Di-tau mass resolution: very poor due to the presence of neutrinos in the final state
  - Visible mass (mass of visible objects)
  - "Missing Mass Calculator" (MMC):

Constrain unknown neutrino momenta using τ decay kinematics

NIM A654 (2011) 481



- Z → τ τ : very important background source



"τ-embedded" Z → μμ data events: select Z→μμ events from data and replace μ with a simulated τ

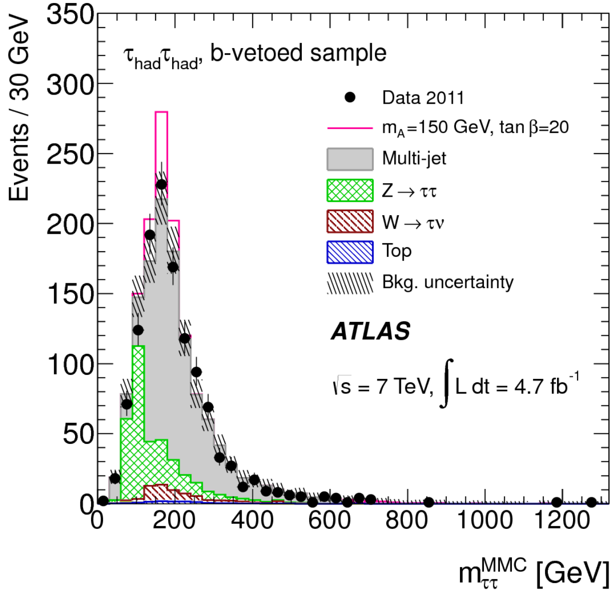
h/H/A → τ τ

JHEP02(2013)095

τ(had)τ(had)

2 τhad p_T > 30/45 GeV; Opposite sign; MET > 25 GeV

"b-vetoed" sample: leading jet (p_T >20GeV) is a b-jet; Leading tau p_T > 60 GeV	**"b-tagged" sample:** leading jet (p_T >20GeV) is a b-jet; leading (b-)jet p_T < 50 GeV

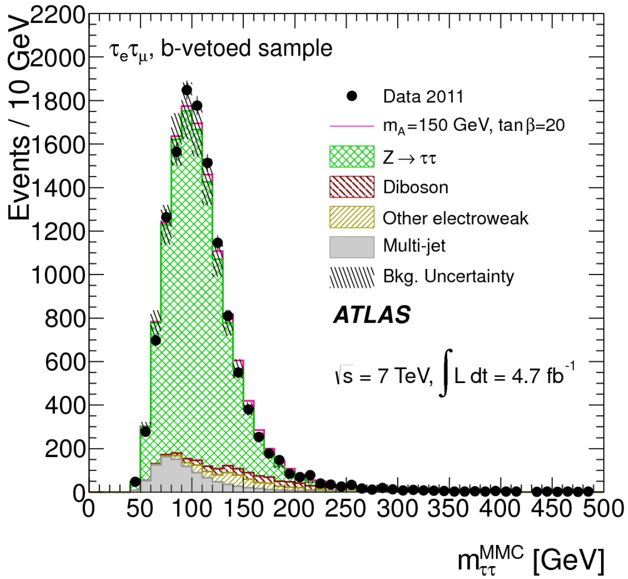


τ(lep)τ(lep) using τ(e)τ(μ) final state

1 isolated e p_T>15-24GeV; **1 isolated μ** p_T>10-20 GeV

Opposite sign; ΔΦ(e,μ)>2; m(e,μ)>30 GeV

"b-vetoed" sample: no b-jets (p_T >20GeV) + topological and other cuts	**"b-tagged" sample:** exactly 1 b-jet (p_T >20GeV) + topological and other cuts



h/H/A → τ τ

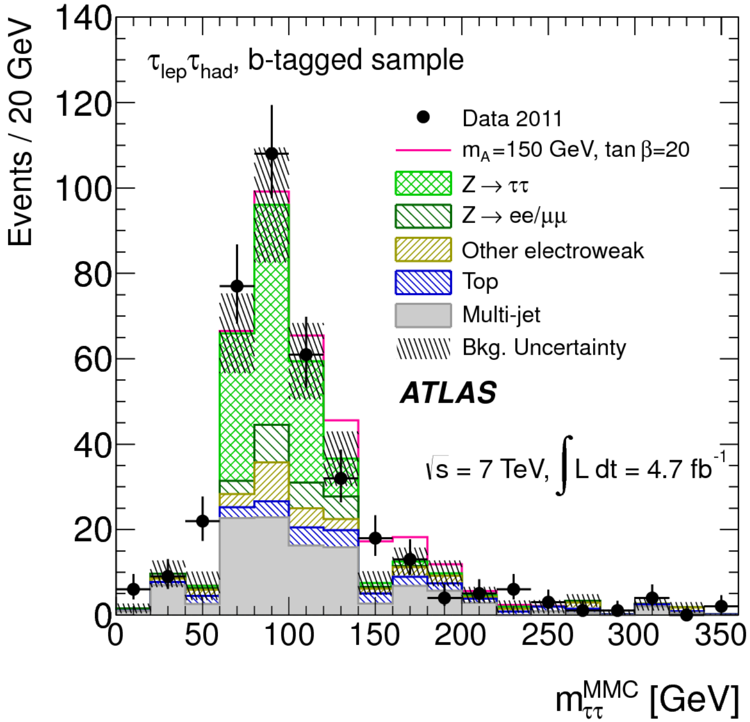
JHEP02(2013)095

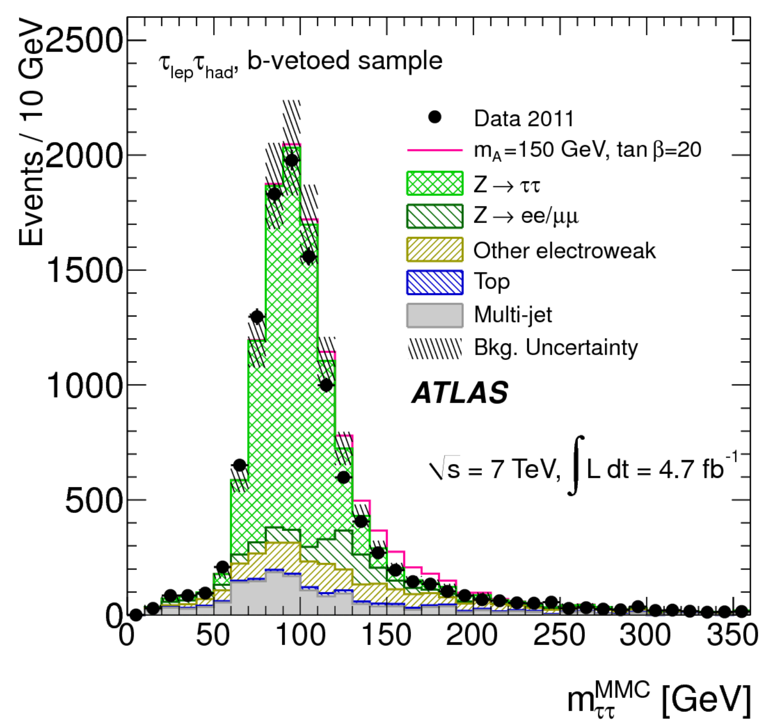
τ(e/μ)τ(had)

isolated e / μ with p_T >25/20 GeV; **τhad** with p_T > 20 GeV;

Opposite sign; M_T < 30 GeV

"b-tagged" sample: leading jet (p_T >20GeV) is a b-jet; Leading (b-)jet p_T < 50 GeV	**"b-vetoed" sample:** leading jet (p_T >20GeV) not a b-jet; MET > 20 GeV





JHEP02(2013)095

h/H/A → μ μ

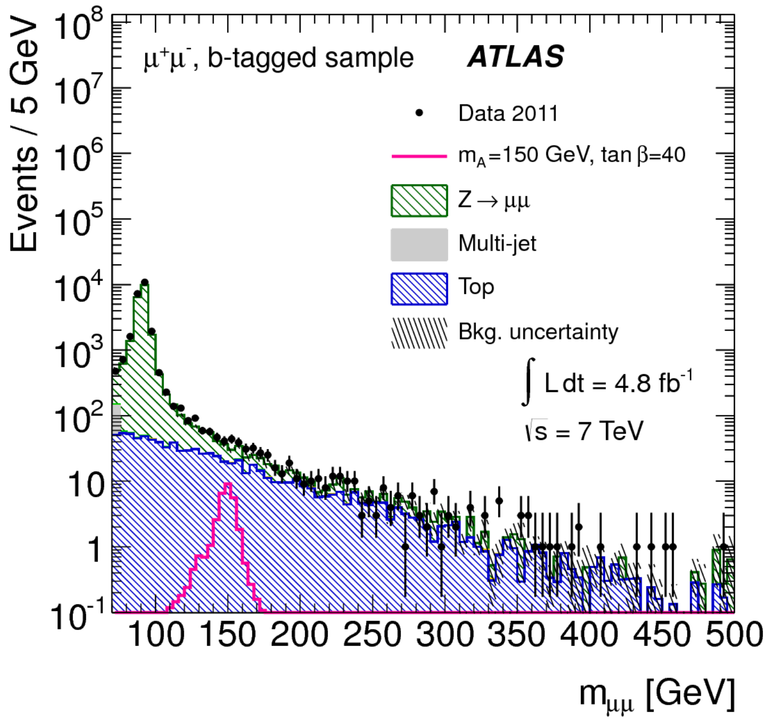
H → μμ

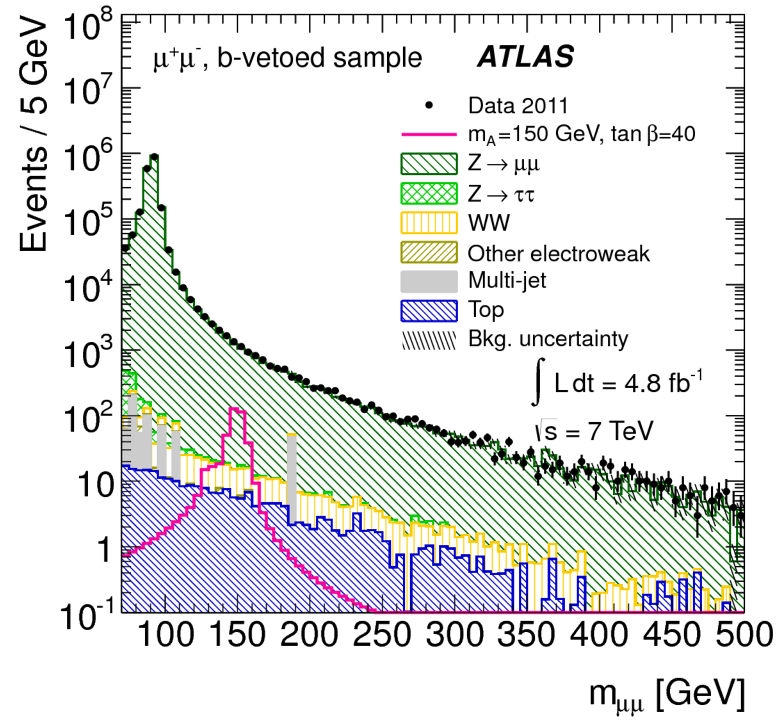
2 μ with p_T > 15/20 GeV; Opposite sign; MET < 40 GeV; m(μμ) > 70 GeV

"b-tagged" sample: at least one b-jet (p_T >20GeV) | **"b-vetoed" sample:** no b-jet (p_T >20GeV)

Bkg model: (Z/γ* interference) ʘ (Gaussian resolution); ʘ = convolution operator

Signal model: (Breit-Wigner) ʘ (Gaussian resolution)+Landau





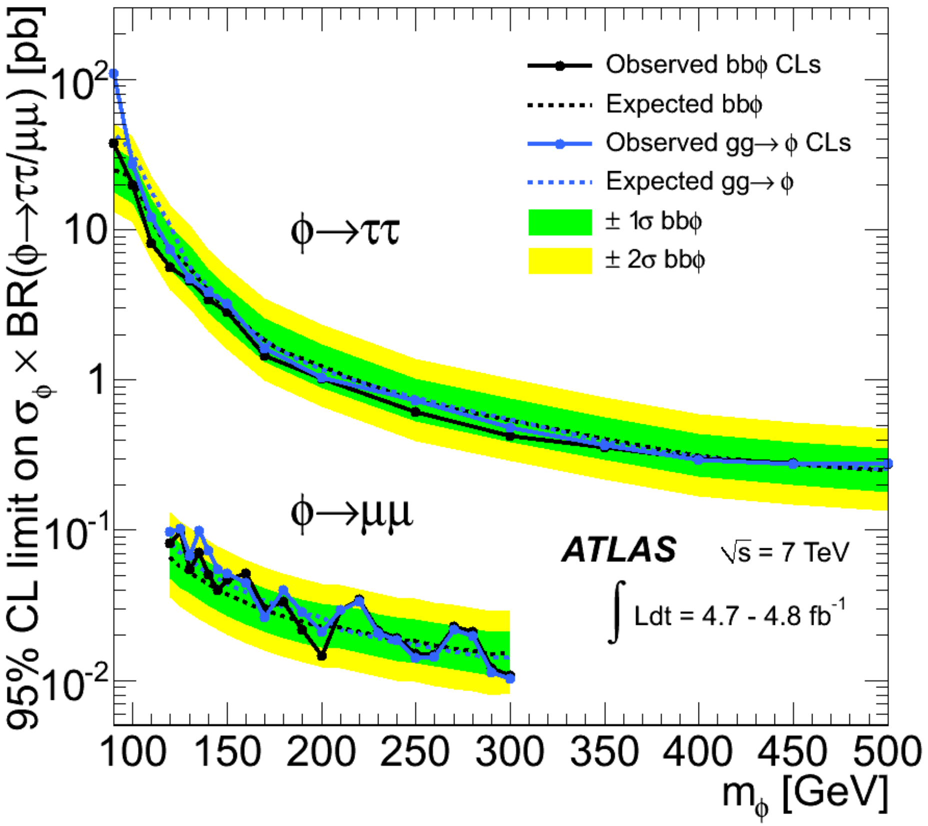
Simulated backgrounds are shown here only for demonstration: not used in the final result

h/H/A → τ τ / μ μ

- Exclusion Limits: all channels combined

JHEP02(2013)095

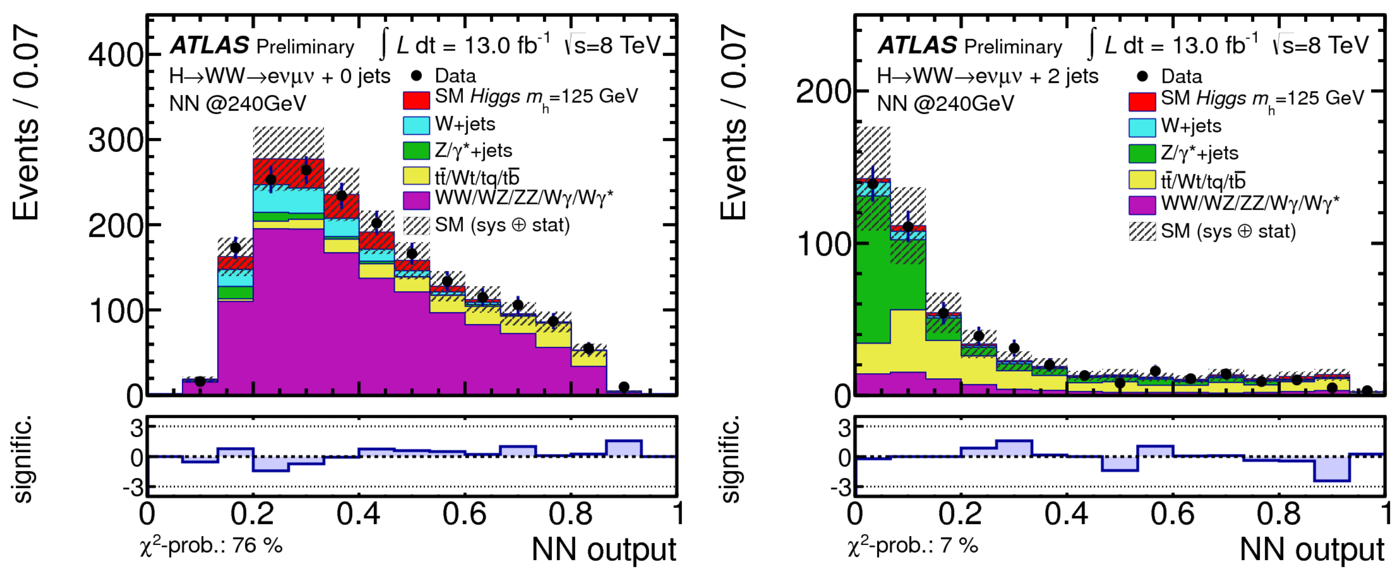
Limit on σ BR(φ → ττ)



H → WW

- Higgs decays to WW are important in 2HDM at certain parts of the parameter space
- First dedicated 2HDM search in H → WW → eν μν targeting gluon-fusion & VBF production; m_h = 125 GeV

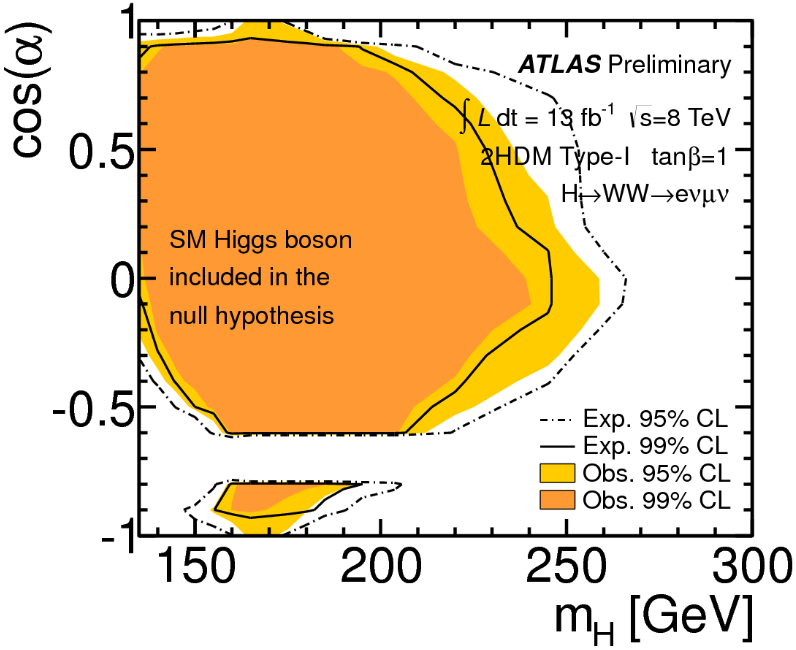
H → W(→ μν) W(→ eν)
isolated e + μ, p_T > 25 (15) GeV leading (subleading)
0 and 2 jet categories
Use of Neural Networks to separate signal from background

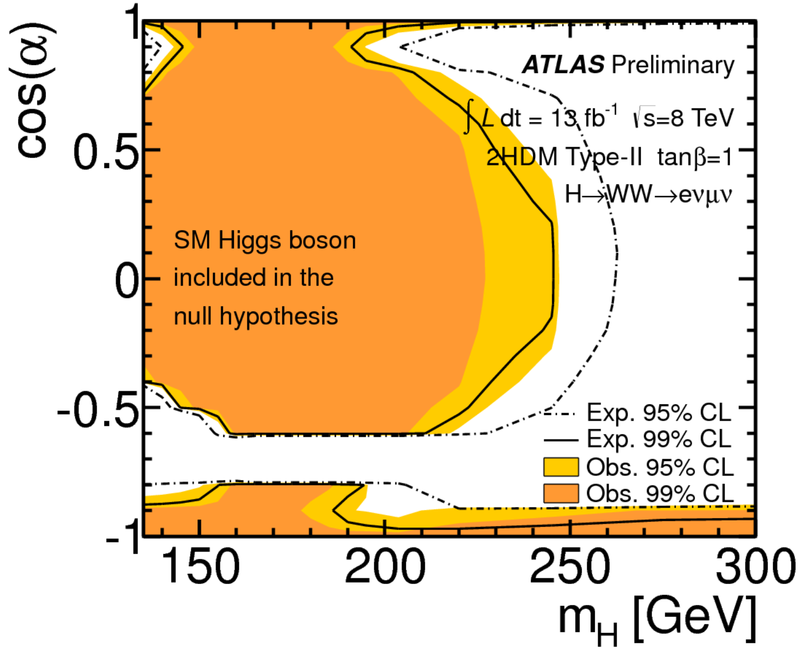


ATLAS-CONF-2013-027

$H \rightarrow WW$

Search interpretation in 2HDM type-I and type-II parameter space

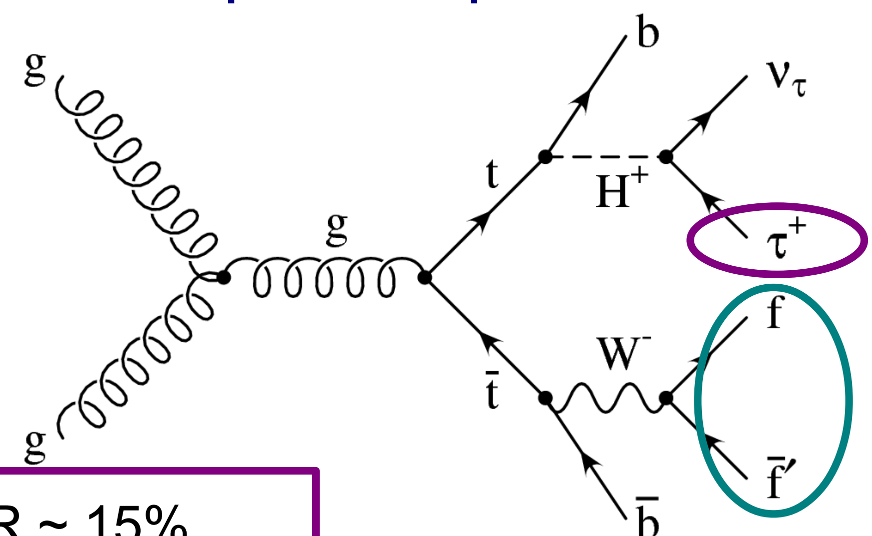




ATLAS-CONF-2013-027

Looking for a charged scalar particle

- Search for a light ($m < m_{top}$) charged scalar particle produced in top decays and decaying:
  $H^{\pm} \rightarrow \tau^{\pm}\nu$ / cs



Channel topology organized according to W and tau decay

$H^{\pm} \rightarrow \tau^{\pm}\nu$

τ(lep)+W(→lν):	$tt \rightarrow bbWH \rightarrow bb\ (l\nu)\ (\tau_{lep}\nu)$	BR ~ 15%
τ(had)+W(→lν):	$tt \rightarrow bbWH \rightarrow bb\ (l\nu)\ (\tau_{had}\nu)$	BR ~ 14%
τ(had)+W(→jets):	$tt \rightarrow bbWH \rightarrow bb\ (qq)\ (\tau_{had}\nu)$	BR ~ 46%
τ(lep)+W(→jets):	$tt \rightarrow bbWH \rightarrow bb\ (qq)\ (\tau_{lep}\nu)$	BR ~ 25%

τ(lep) = τ(e) or τ(μ)

$H^{\pm} \rightarrow cs$

$H^{+}(\rightarrow cs)+W(\rightarrow l\nu)$: $tt \rightarrow bbWH \rightarrow bb\ (l\nu)\ (cs)$

$H^{\pm} \rightarrow \tau^{\pm}\nu$ search

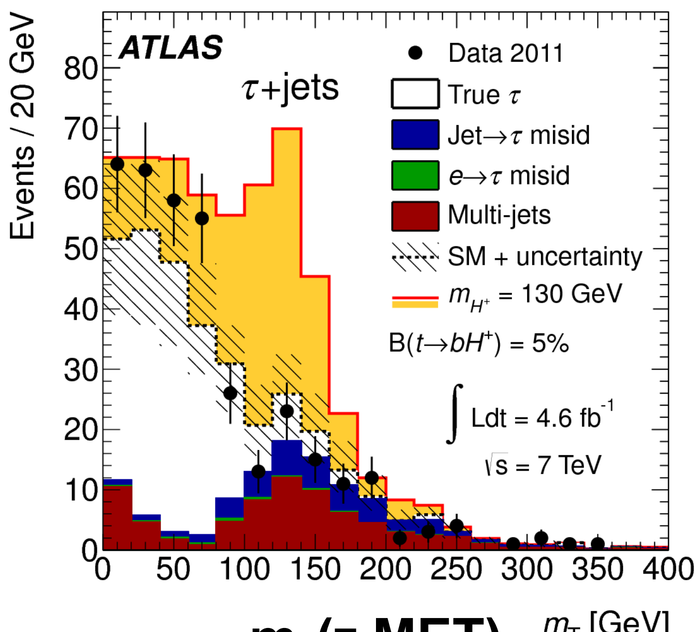
JHEP 1206 (2012) 039

ATLAS $H^{\pm} \rightarrow \tau^{\pm}\nu$ search uses 3 channels:

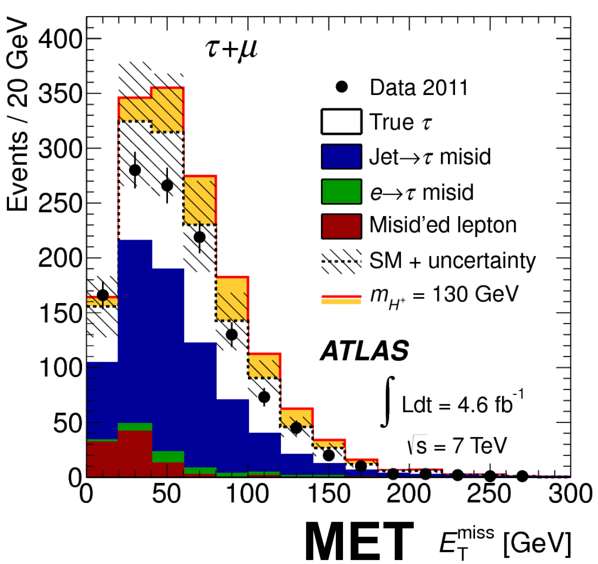
τ(had)+W(→jets) **τ(had)+W(→lν)** **τ(lep)+W(→jets)**

τ(had) + W(→ jets)
1 τ_{had} with pT > 40 GeV
At least 4 jets (pT>20 GeV) with at least 1 b-tagged
MET > 65 GeV (tighter at high Σ p_T(tracks)) 120 GeV < m(jjb) < 240 GeV

Most sensitive channel, but the absence of a light lepton makes triggering on these events not trivial: tau + MET trigger



m_T(τ,MET)



MET

τ(had) + W(→ lν)
1 isolated e/μ, pT > 25/20 GeV; 1 τ_{had} with pT > 20 GeV
At least 2 jets (p_T>20 GeV), with at least 1 b-tagged
vertex Σp_T > 100 GeV

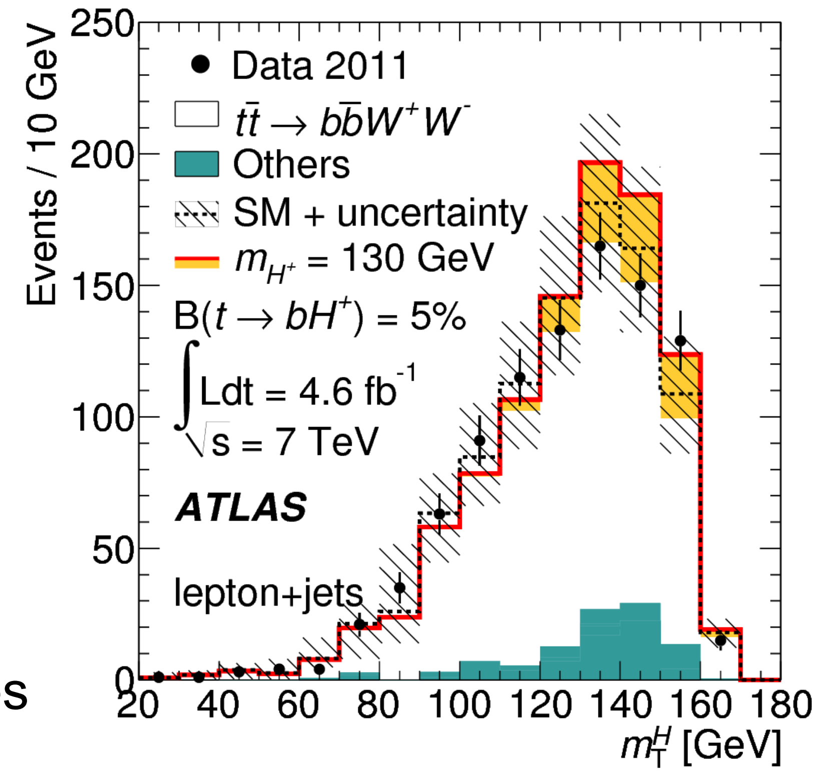
$H^\pm \to \tau^\pm \nu$ search

JHEP 1206 (2012) 039

ATLAS $H^\pm \to \tau^\pm \nu$ search uses 3 channels:

τ(had)+W(→jets) **τ(had)+W(→lν)** **τ(lep)+W(→jets)**

τ(lep) + W(→ jets)
1 isolated e/μ, $p_T > 25/20$ GeV
At least 4 jets (p_T>20 GeV) with exactly 2 b-tagged
MET > 40 GeV (tighter if Δφ(lepton,MET) small) $\cos\theta^*_l < -0.6$; m_T(lepton,MET)<60 GeV



Very challenging to separate signal from tt → bbWW → bb+jj+lν (main background)

kinematic fit to associate b-jets to the top candidates

$$\cos\theta^*_l = \frac{2m^2_{bl}}{m^2_{top} - m^2_W} - 1$$

b-jet+ charged lepton invariant mass

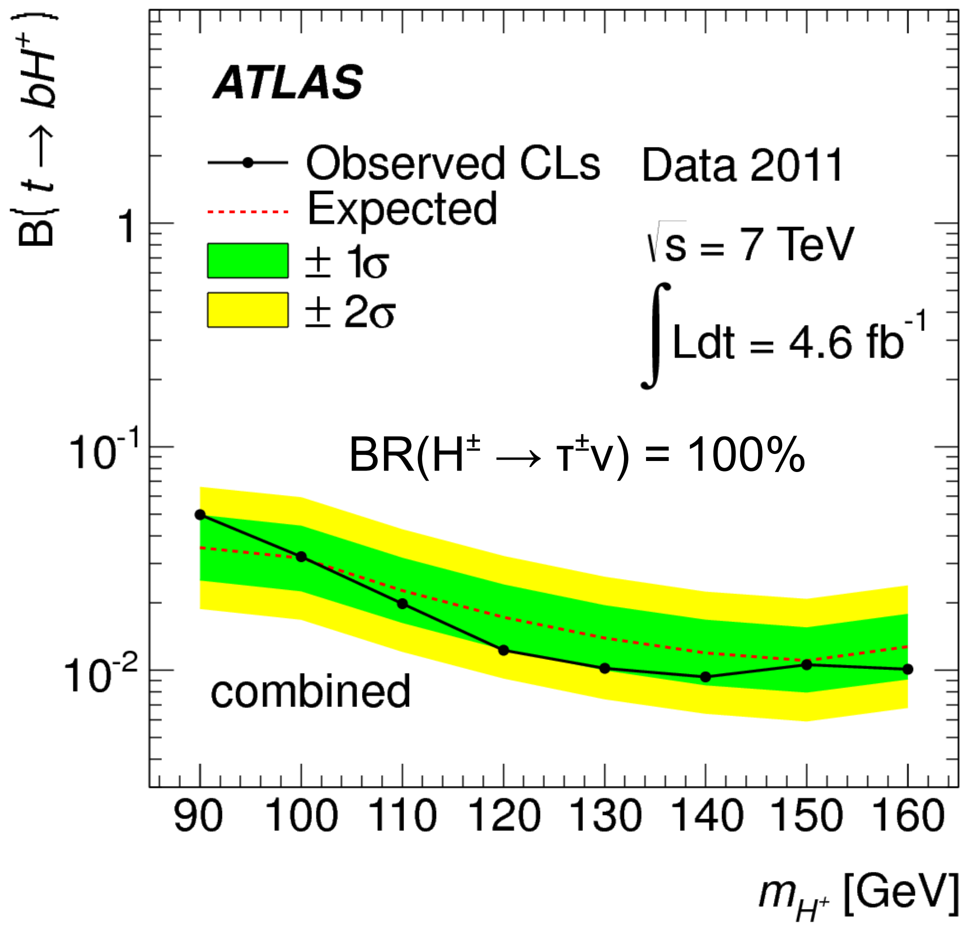
$$(m_T^H)^2 = \left(\sqrt{m^2_{top} + (\vec{p_T}^l + \vec{p_T}^b + \vec{p_T}^{miss})^2} - p_T^b\right)^2 - \left(\vec{p_T}^l + \vec{p_T}^{miss}\right)^2.$$

Higgs transverse mass

$H^{\pm} \rightarrow \tau^{\pm}\nu$ search

JHEP 1206 (2012) 039

Branching fraction limits



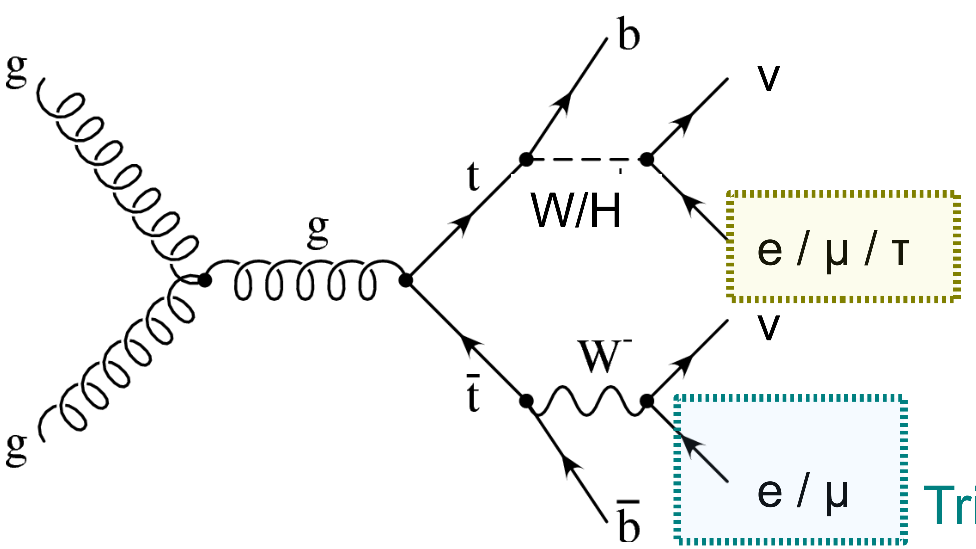
$H^{\pm} \rightarrow \tau^{\pm}\nu$ search with the "Ratio" method

- If a $H^{\pm}$ boson is produced in top decays its preferred decay mode to $\tau\nu$ can be observed as **lepton flavour universality violation**:

In the absence of new physics $R_e = 1$ and $R_\mu = 1$ to a very good approximation

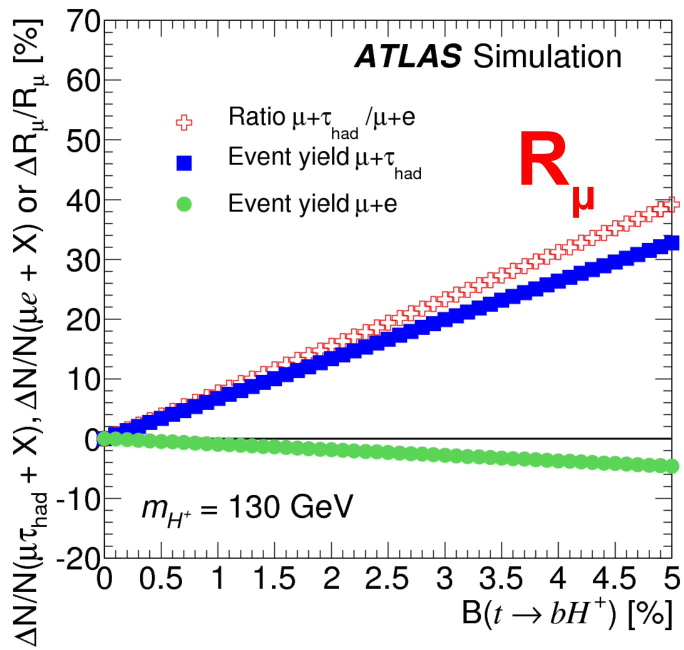
JHEP03(2013)076

$$R_e = \frac{\mathcal{N}(e + \tau_{\text{had}})}{\mathcal{N}(e + \mu)} \qquad R_\mu = \frac{\mathcal{N}(\mu + \tau_{\text{had}})}{\mathcal{N}(\mu + e)}$$



Probe lepton

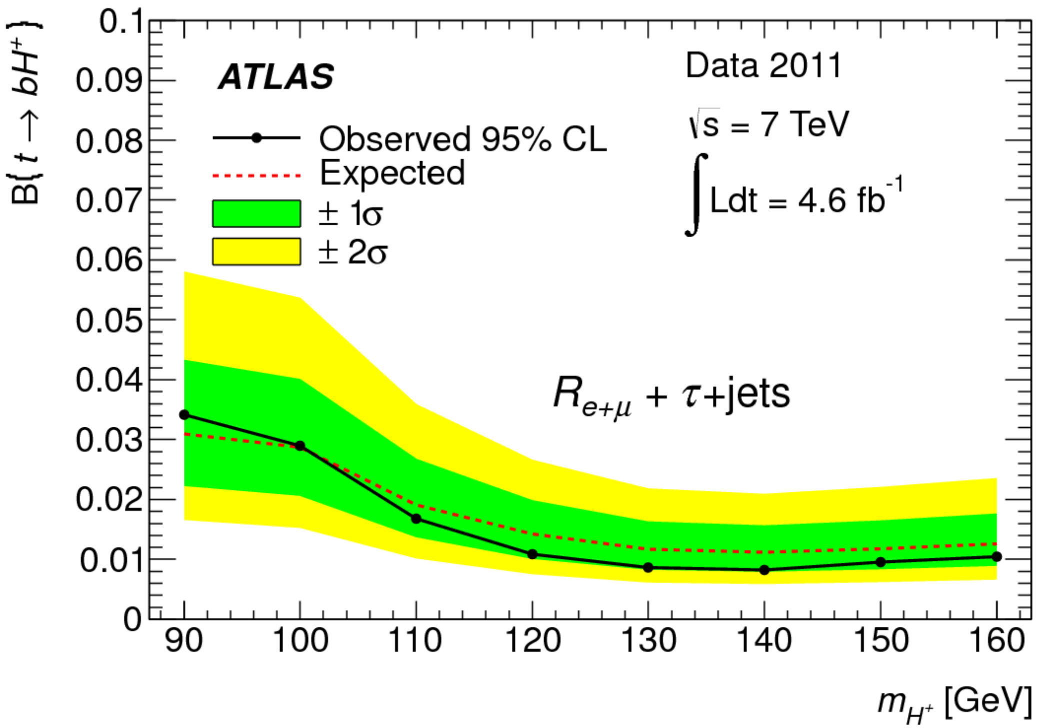
Trigger (event tagger)



$H^{\pm} \rightarrow \tau^{\pm}\nu$ search with the "Ratio" method

JHEP03(2013)076

$H^{\pm} \rightarrow \tau\nu$ using lepton flavour universality violation: $W(\rightarrow l\nu) + l'$ or τ
1 isolated e/μ, $p_T > 25$ GeV; MET > 40 GeV
At least 2 jets (p_T>20 GeV), with exactly 2 b-tagged
(1 τ_{had} $p_T > 20$ GeV and no other lepton) **or** 1 additional lepton p_T>25 GeV of a different flavour wrt the lepton tagged the event

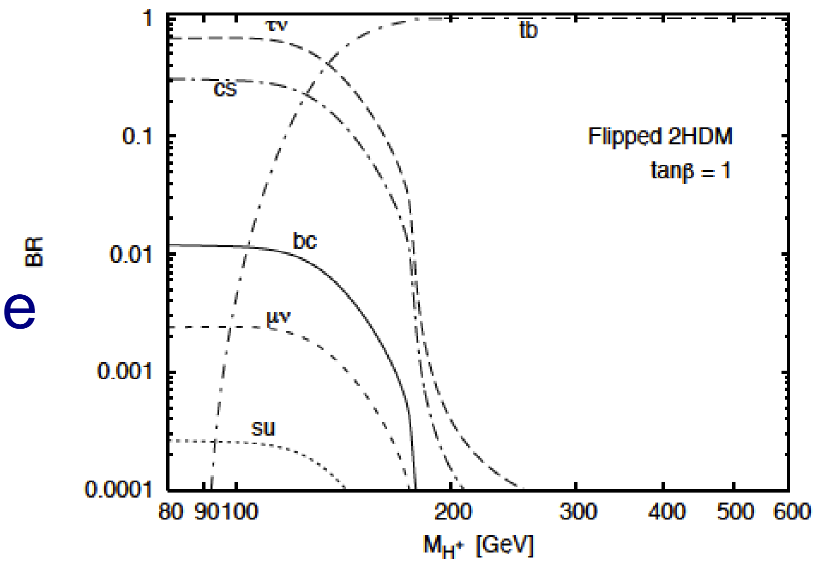


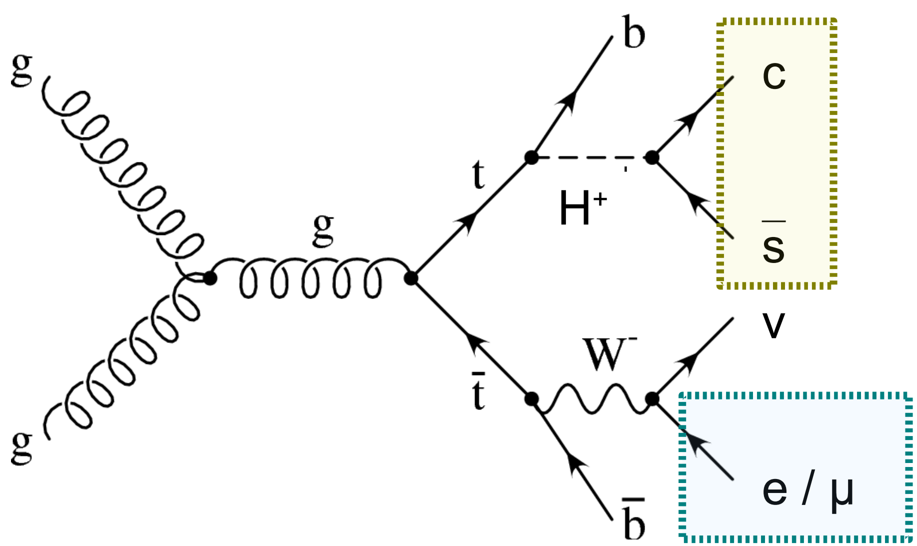
The result is combined with the direct $H^{\pm} \rightarrow \tau\nu$ search in the τ(had)+W(→jets) channel

$H^{\pm} \rightarrow cs$

- $H^{\pm} \rightarrow cs$ is important for various 2HDM flavours
- The ATLAS search looks for this decay in top pair production in which one of the tops decays leptonically

Logan et al. arXiv:1002.4916



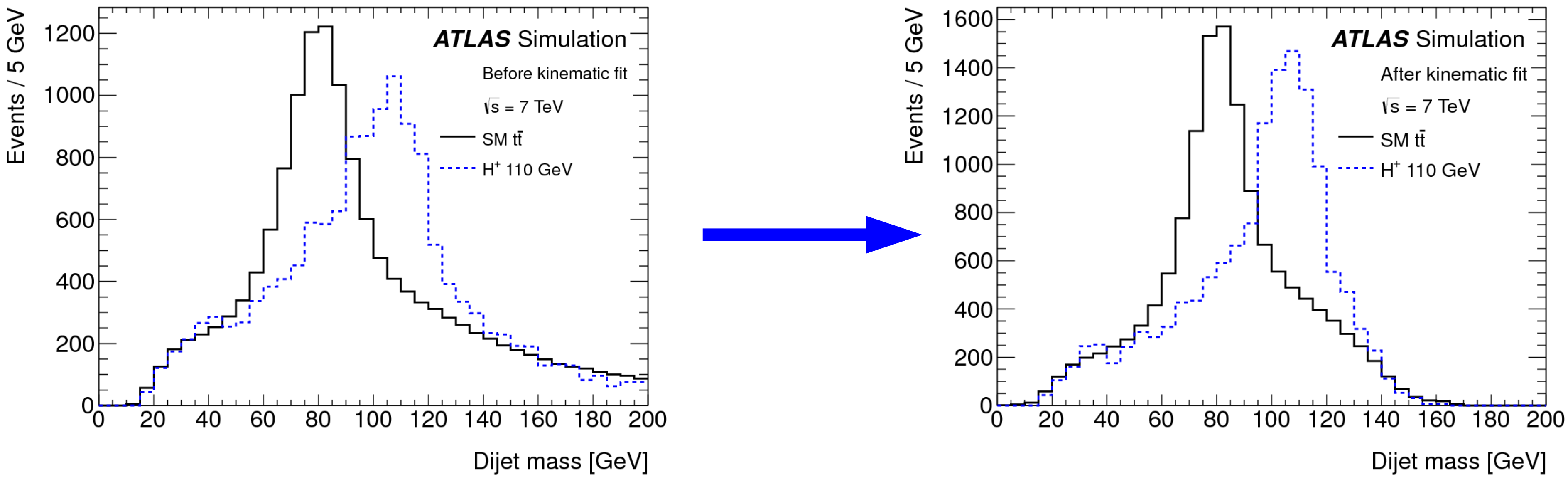


$t\bar{t} \rightarrow bW(\rightarrow l\nu) + bH^{\pm}(\rightarrow cs)$
1 isolated e/μ, $p_T > 25/20$ GeV
At least 4 jets $p_T > 25$ GeV and at least 2 of these jets are b-tagged
MET > 20 (30) GeV in the μ (e) channel; $m_T > 30$ GeV (e channel) and m_T + MET > 60 GeV (μ channel)

Eur. Phys. J. C (2013) 73:2465

$H^{\pm} \rightarrow cs$

- Discriminating variable is the invariant mass of the 2 jets from the Higgs decay
  - Kinematic fitter is used to improve mass resolution

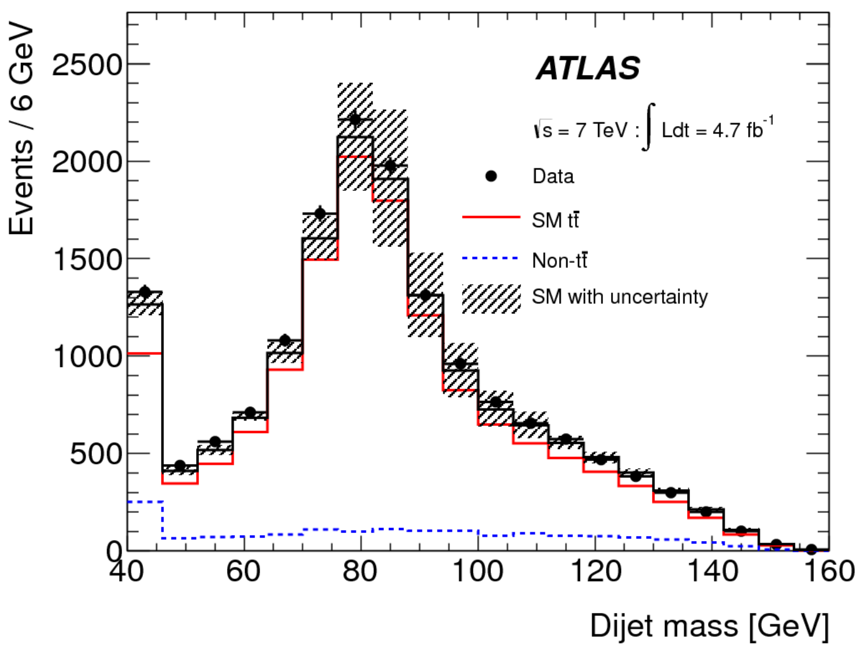


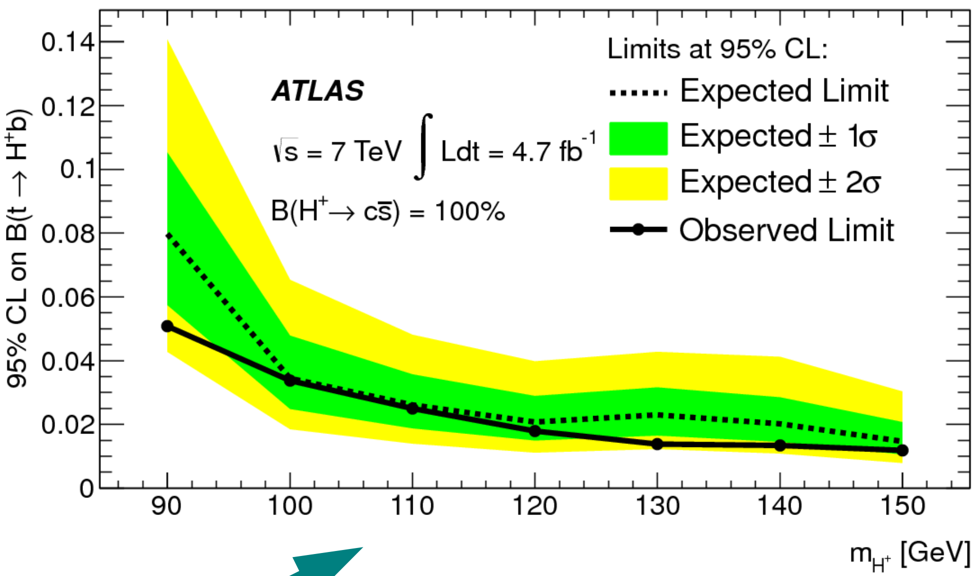
~ 20 – 30% improvement in mass resolution

Eur. Phys. J. C (2013) 73:2465

$H^{\pm} \rightarrow cs$

Final discriminant in data and SM expectation after full selection:





Best available limits so far in this process!!!

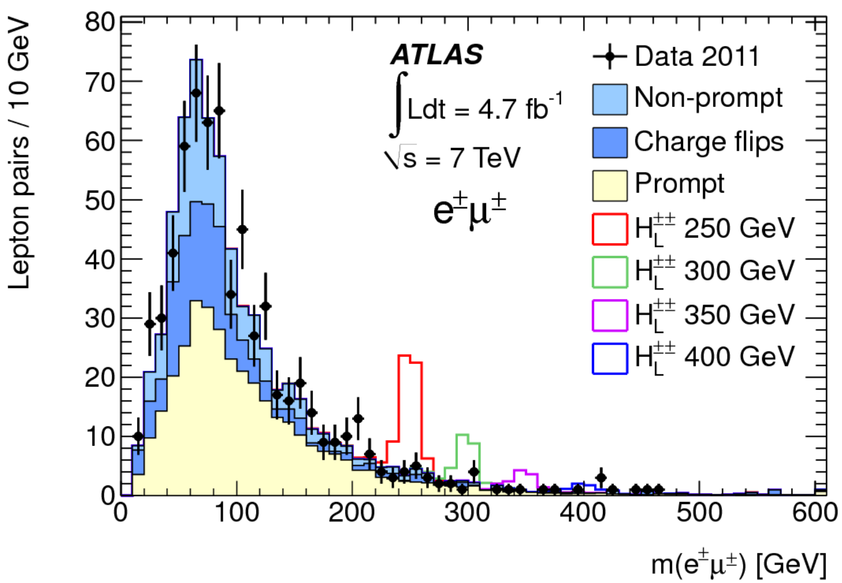
Eur. Phys. J. C (2013) 73:2465

Doubly Charged Scalars

- Beyond the 2HDM, the addition of triplet is feasible, under some assumptions in order to keep ρ ~ 1
  - Doubly charged scalars are contained in such triplets
- ATLAS search for $H^{\pm\pm}$

Eur.Phys.J. C72 (2012) 2244

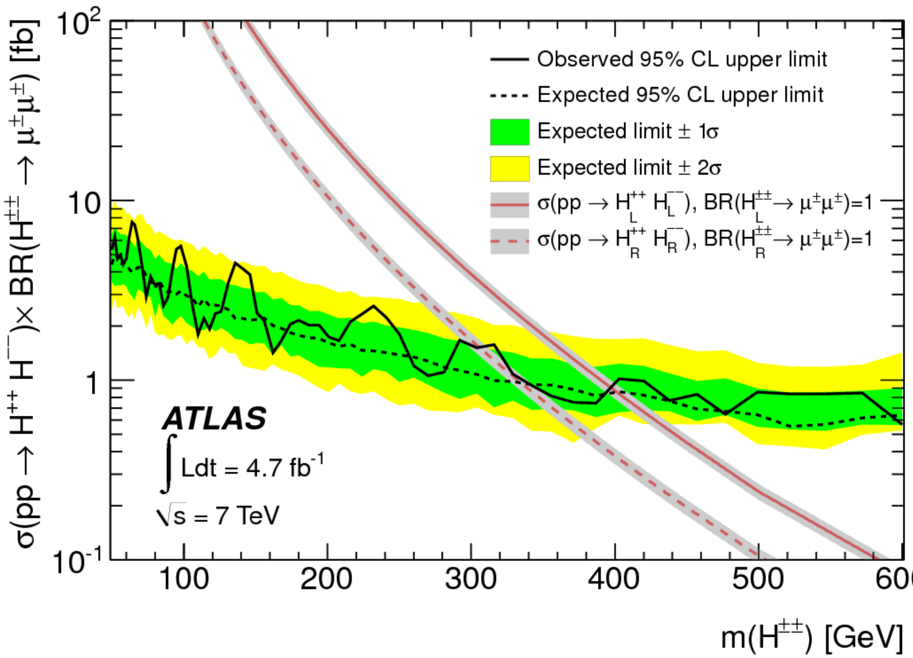
◇ $H^{\pm\pm}$ production in pairs through Z/γ*
◇ $H^{\pm\pm}$ assumed decay modes:
$H^{\pm\pm} \rightarrow e^{\pm}e^{\pm}, \mu^{\pm}\mu^{\pm}, e^{\pm}\mu^{\pm}$
◇ Select pairs of same charge particles in the events: ee, μμ, eμ
◇ Look for an excess in the same sign di-lepton invariant mass distribution

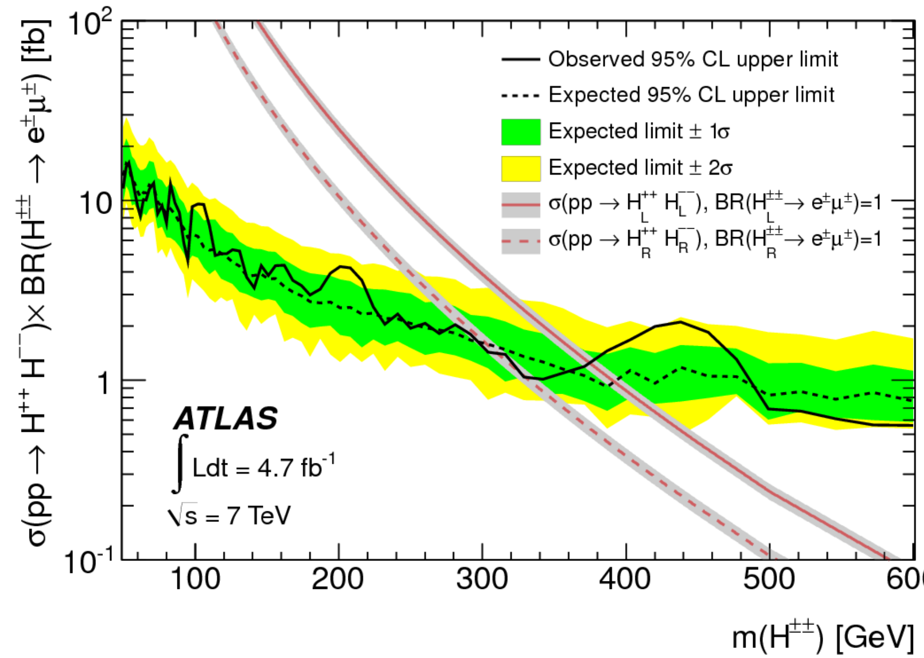


Doubly Charged Scalars

- Cross section & BR limits and comparison to production cross section of left-handed and right-handed $H^{\pm\pm}$

Eur.Phys.J. C72 (2012) 2244





Higgs as a link to New Sectors

- The Higgs sector of the SM has unique properties
  - The Higgs doublet Φ is such that $\Phi^\dagger\Phi$ is a singlet of dimension 2
  - Couplings of the type $\Phi^\dagger\Phi\varphi^*\varphi$ are just dimension 4 for some new scalar particle φ; $\Phi\varphi\varphi$ can also appear after SSB

The Higgs sector can serve as the connection to a New Sector of Nature!

Such possibilities can be offered by many models:

◇ NMSSM extends the MSSM with an EWK singlet:
decays h → a1 a1 → 4 γ are possible ATLAS-CONF-2012-079

◇ "Hidden Valley" models include decays to long lived particles h → π_v π_v

◇ Higgs to invisible ATLAS-CONF-2013-011

PRL 108 (2012) 251801
PLB721 (2013) 32-50

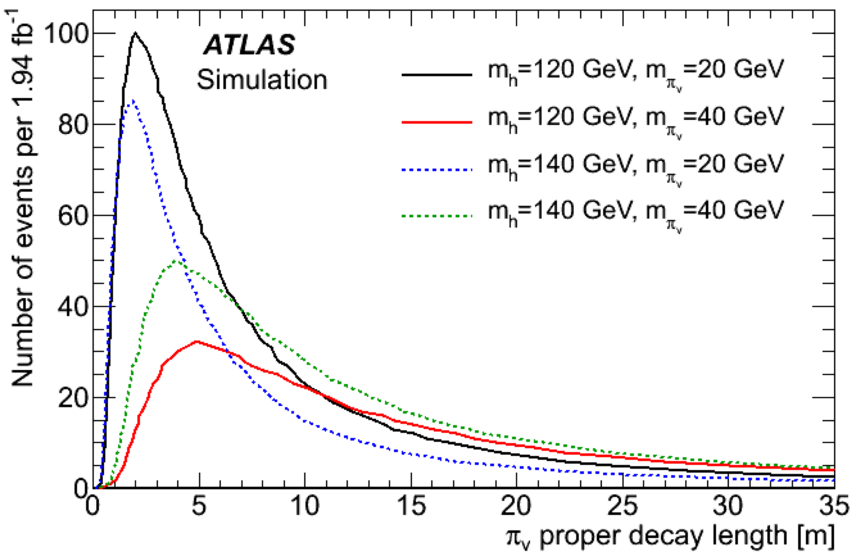
◇ …

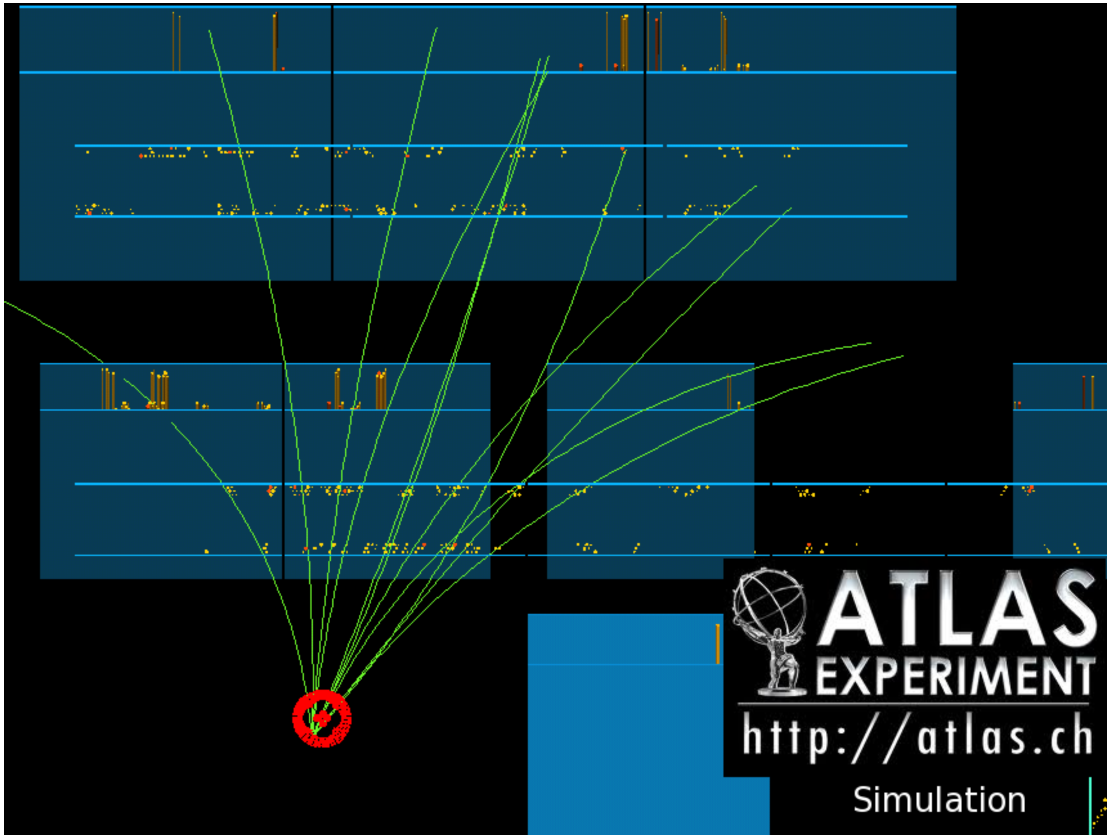
Higgs decaying to long-lived particles

- Higgs decaying to invisible, long-lived "hidden valley" pions π_v, which decay to jets in the outer calorimeter and are detected in the muon system

PRL 108 (2012) 251801

$h \rightarrow \pi_v \pi_v$; $\pi_v \rightarrow bb/cc/\tau\tau$





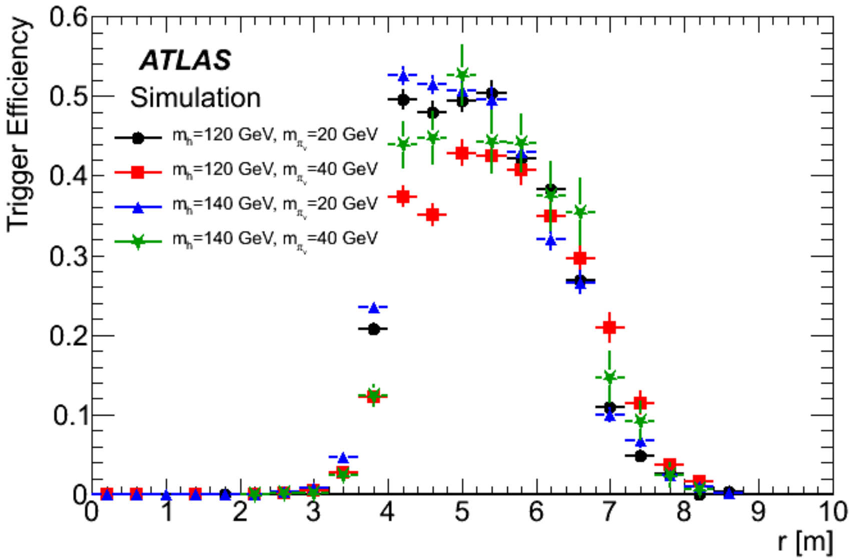
Higgs decaying to long-lived particles

- $h \rightarrow \pi_v \pi_v$; $\pi_v \rightarrow bb/cc/\tau\tau$

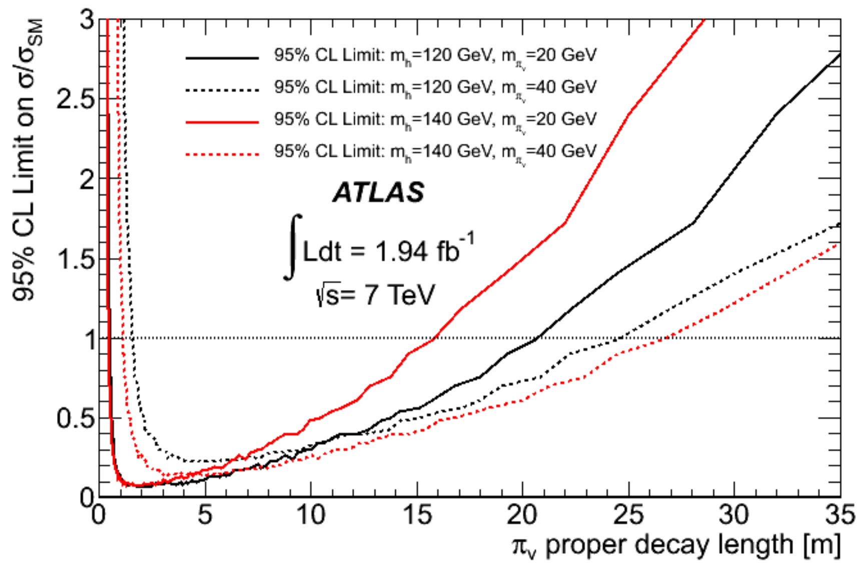
PRL 108 (2012) 251801

Dedicated trigger development to collect candidate events

"hidden valley" model used as benchmark assuming a Higgs produced as in SM, but with a BR ($h \rightarrow \pi_v \pi_v$) = 100%



Radial decay position of long lived particle



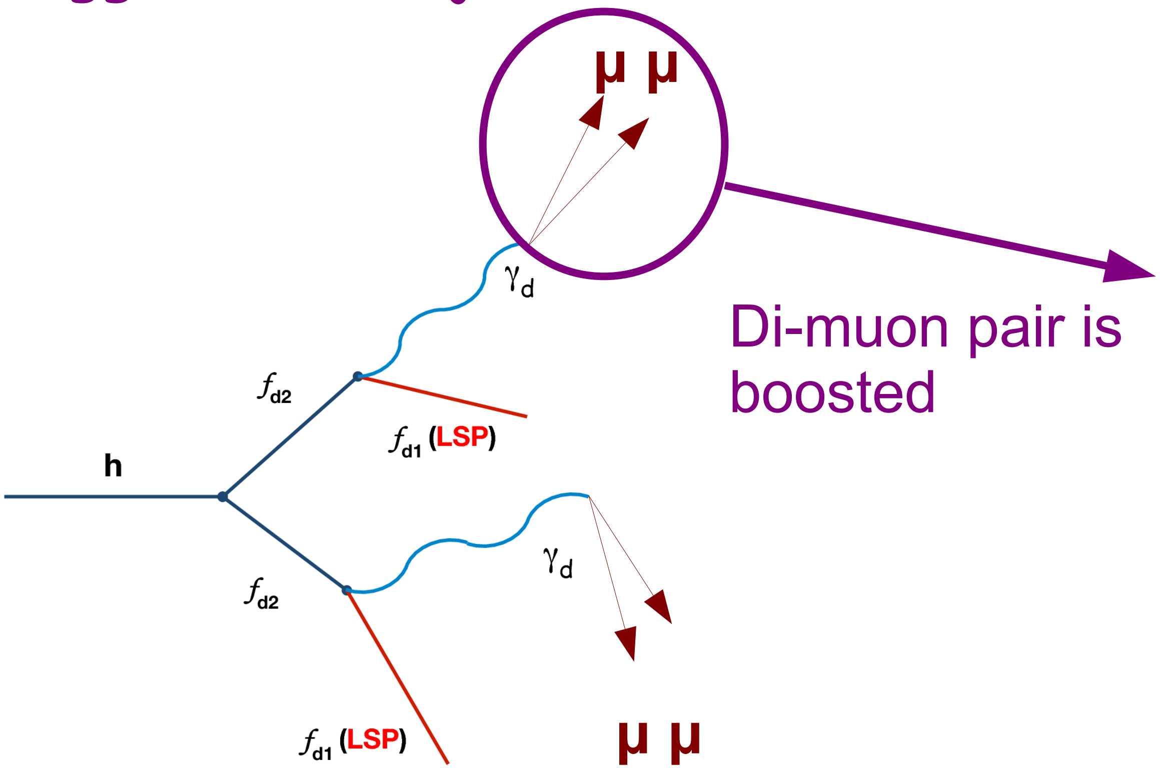
See also arXiv:1305.2284 (submitted to JINST) for a dedicated review on exotic trigger tools for long lived particles

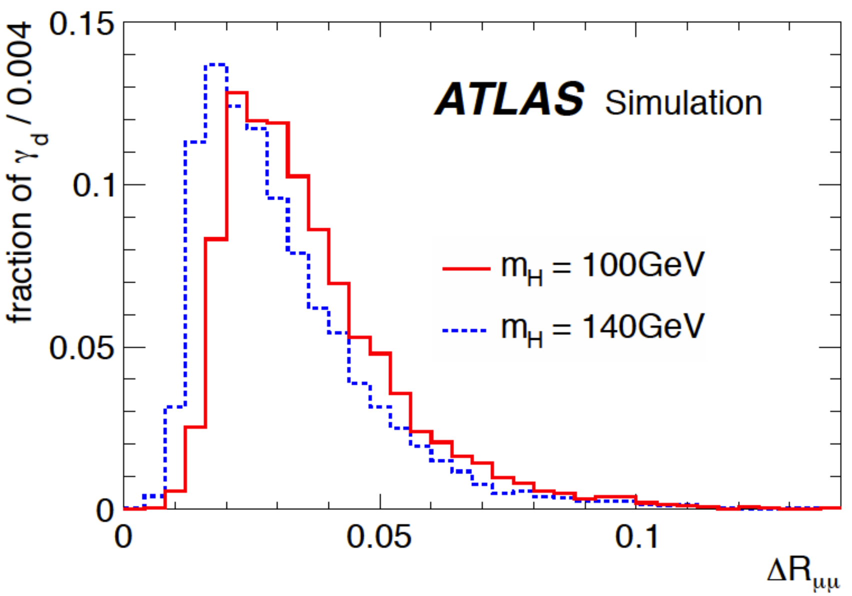
Higgs decaying to long-lived particles

- Higgs decaying to invisible, long-lived particles, which finally produce particles decaying to lepton-jets

Higgs to muon-jets

Phys.Lett. B721 (2013) 32-50





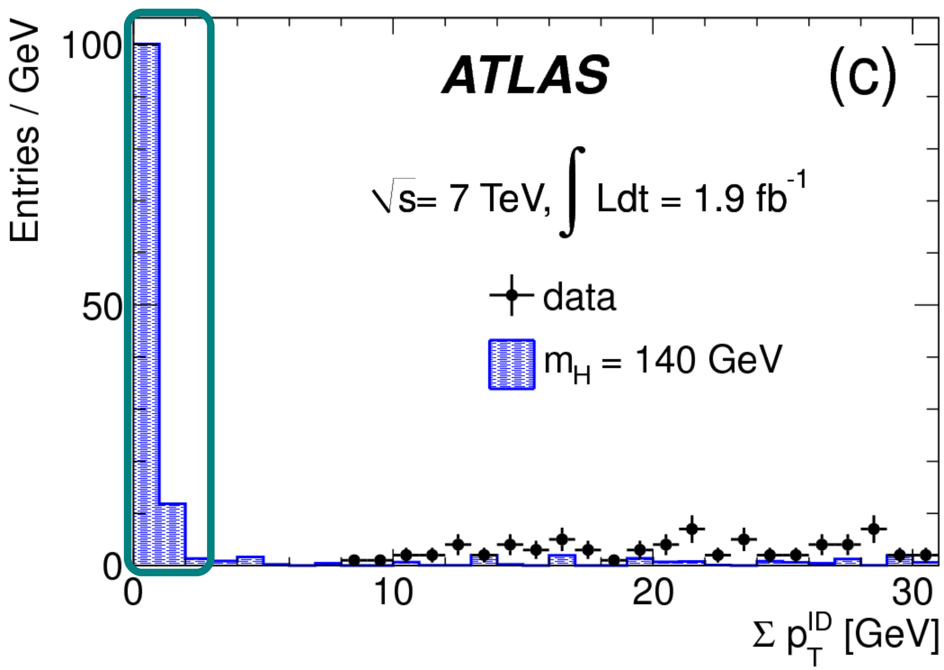
Higgs decaying to long-lived particles

- Higgs to muon jets: results

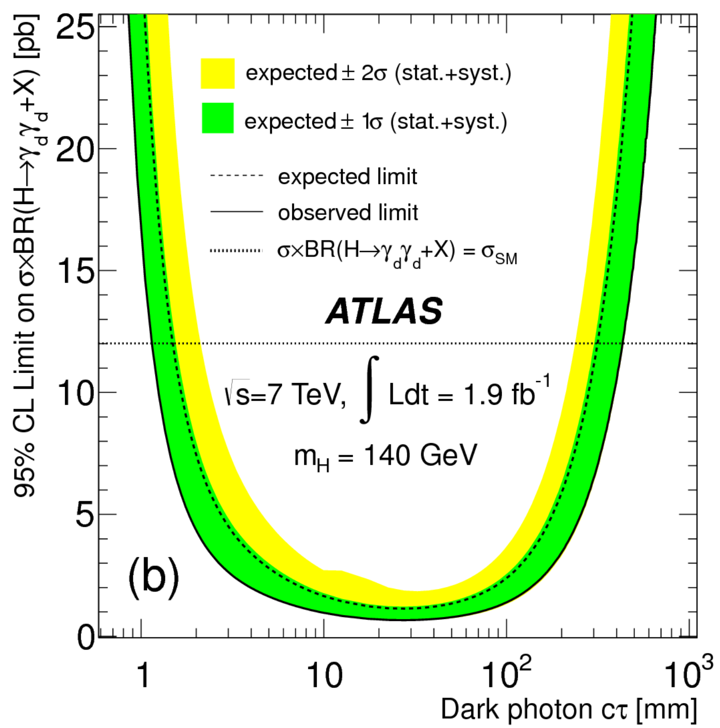
Phys.Lett. B721 (2013) 32-50

Selecting events using a 3-muon trigger with Muon-System-only muons

Signal region



"hidden valley" model used as benchmark assuming BR ($h \rightarrow \gamma_d \gamma_d$ +X) = 100%



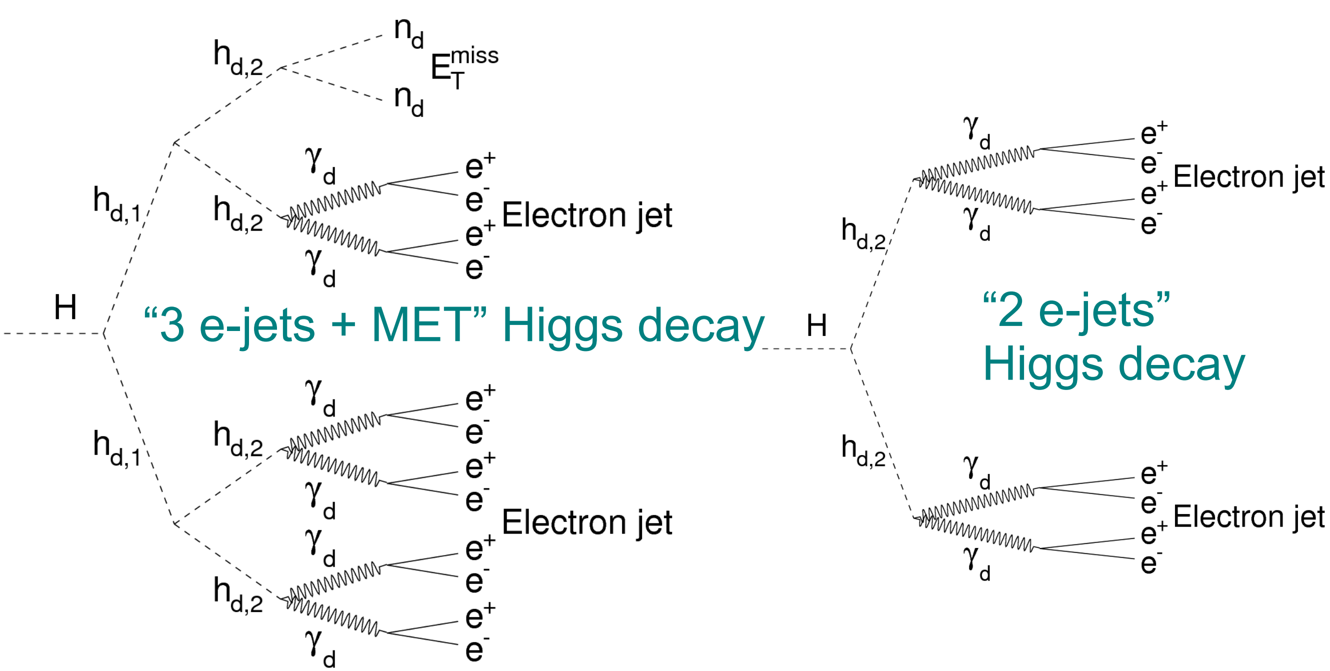
$\Sigma\, p_T$(ID) : Sum of Inner Tracking Detector track pT in DR < 0.4 around the lepton-jet direction

Higgs decaying to prompt electron-jets

- Hidden sector particles aren't necessarily long-lived
- ATLAS has looked for WH production where H decays to prompt electron-jets

New J. Phys. 15 (2013) 043009

Higgs to electron-jets (e-jets)

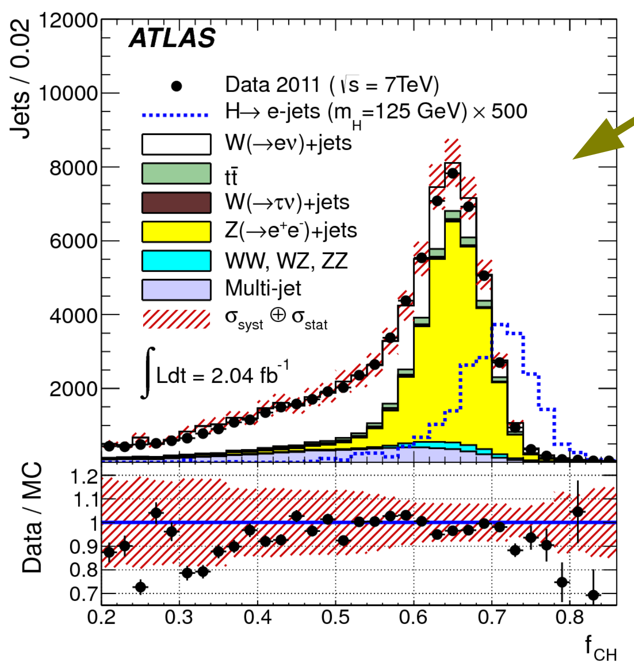


◇ signature: high pT electron or muon from the W decay plus electron-jets in the event

Higgs decaying to prompt electron-jets

- Electron-jets are reconstructed as jets and then requirements on the shower shape and associated track properties are set

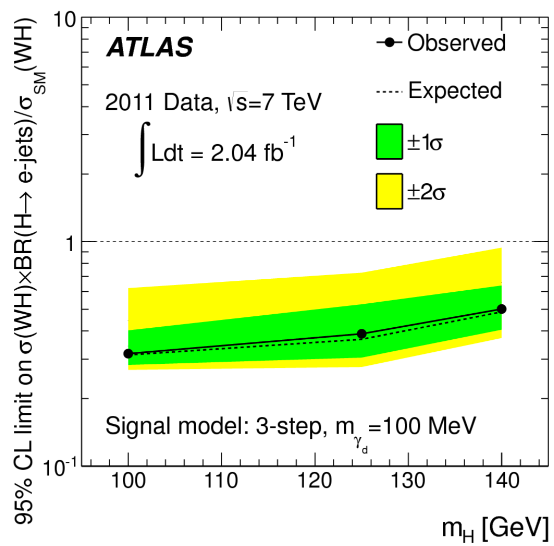
Example: the fraction calorimetric energy associated with tracks is larger in electron-jets than in normal jets from quark/gluon hadronisation



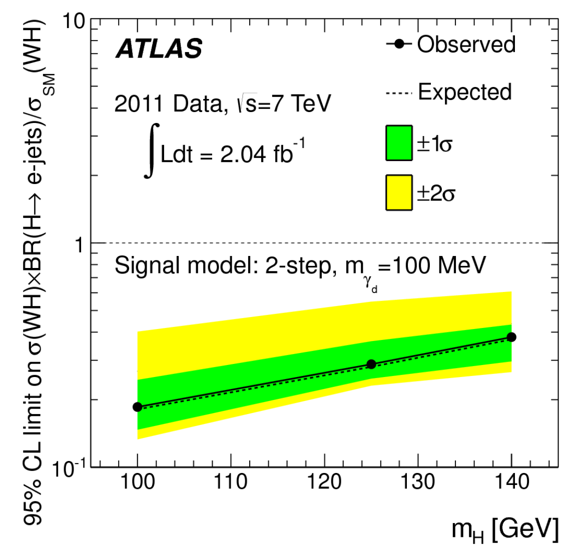
New J. Phys. 15 (2013) 043009

Limits on Higgs production and decay to the two topologies discussed in the previous slide

"3 e-jets + MET" Higgs decay



"2 e-jets" Higgs decay

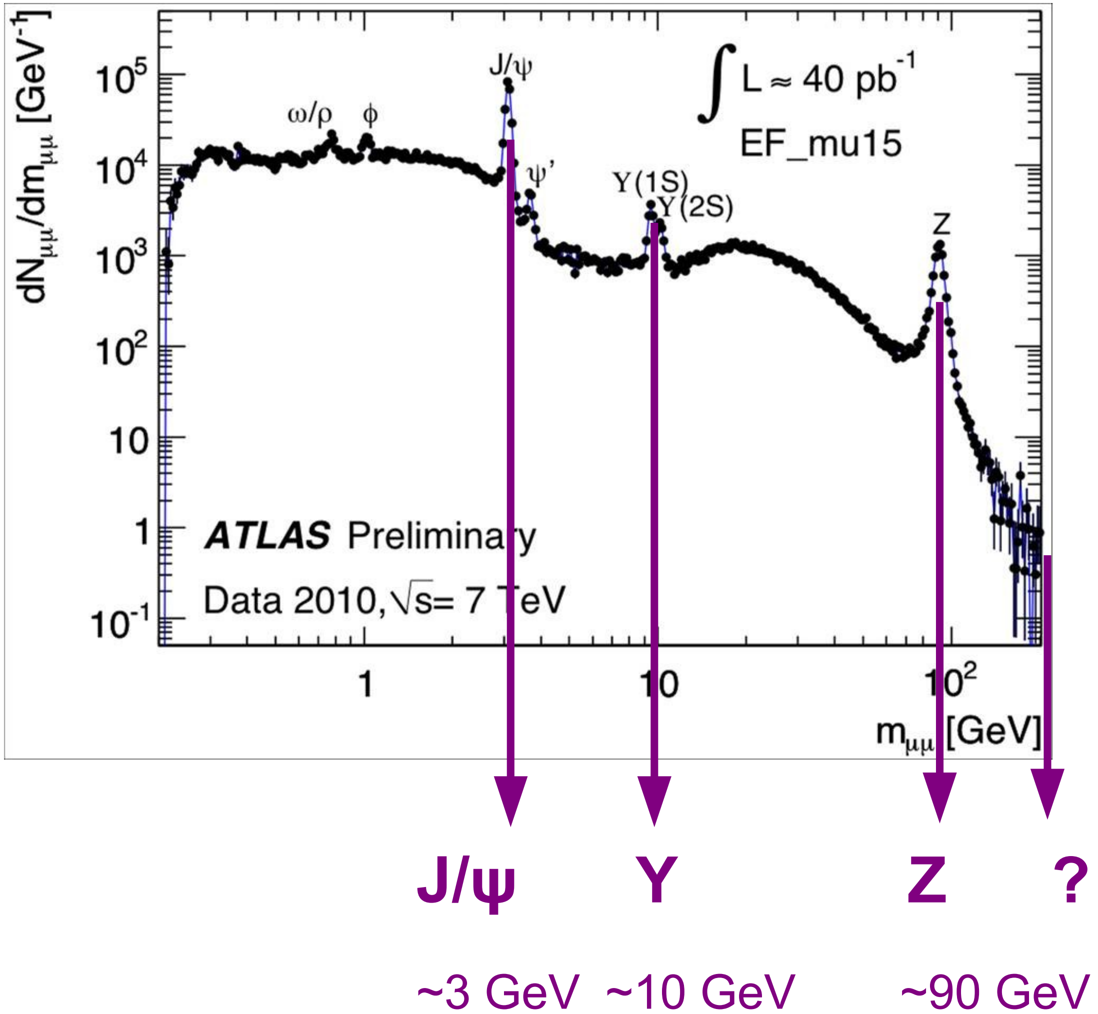


Heavy Resonances

Follow the trend:

◊ at every order of magnitute in di-μ mass a new resonance appears

◊ resonant production of new particles is a striking signature over the tail of the falling spectrum

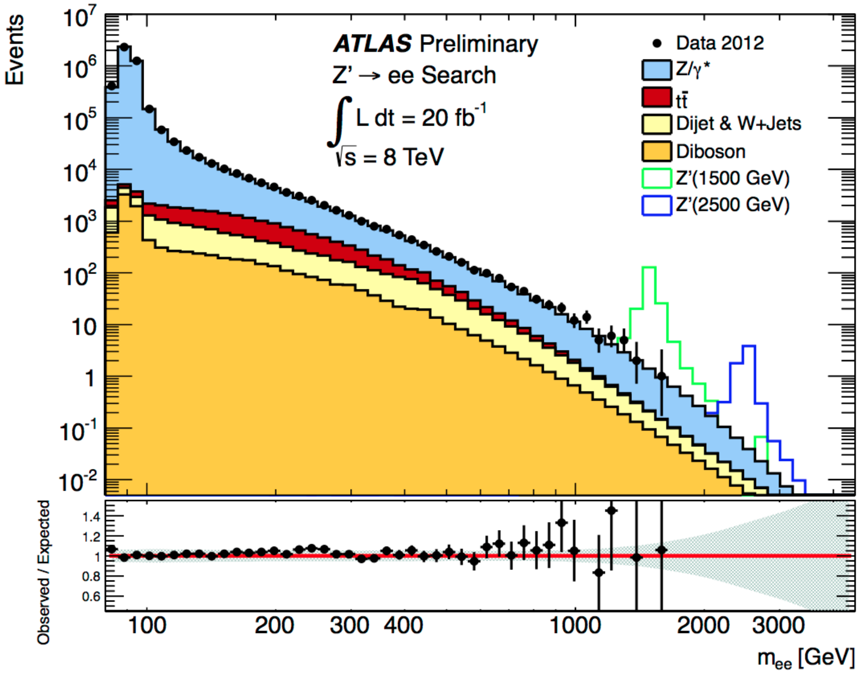


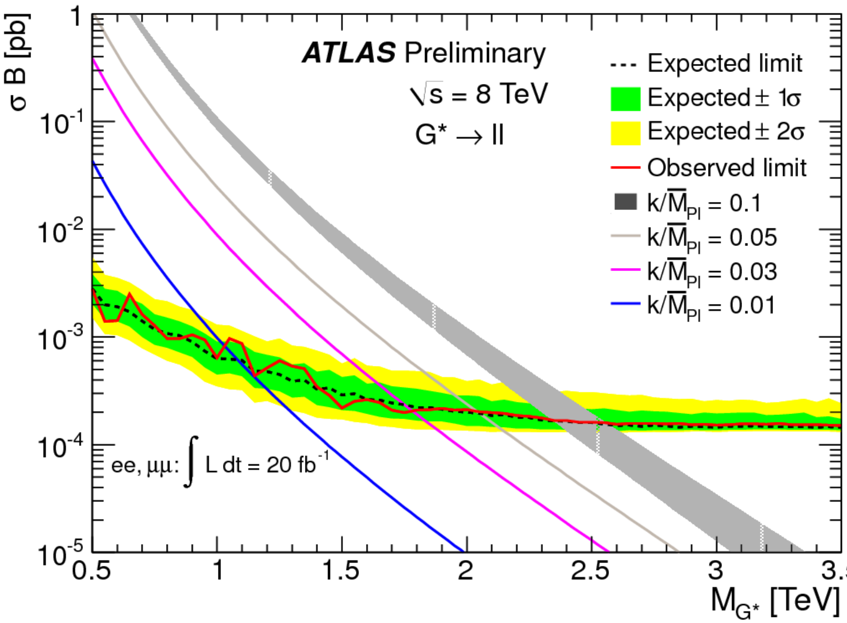
High mass Di-lepton Resonance Search

- $Z' \rightarrow ee / \mu\mu$

ATLAS-CONF-2013-017

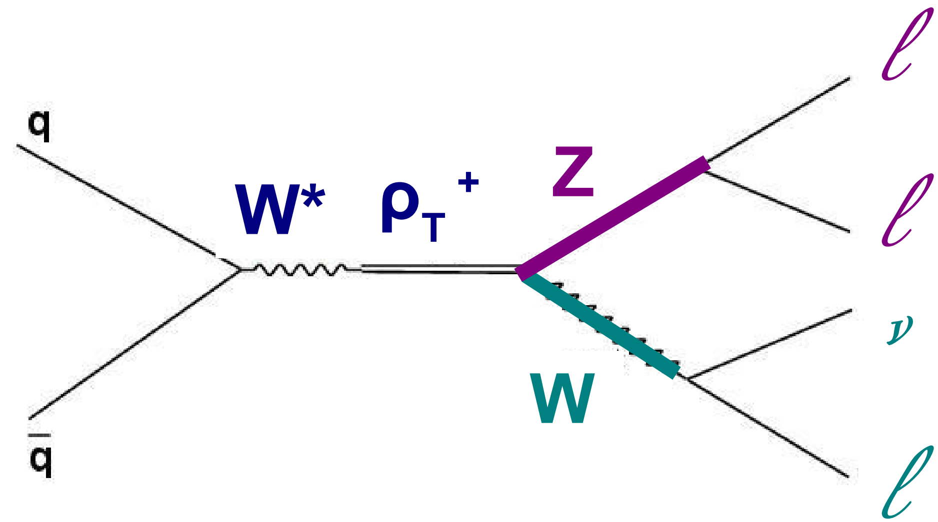
  - Select same flavour lepton pair with highest sum pT
  - Normalize backgrounds to low invariant mass region
  - Data driven estimation for multi-jet and W+jets

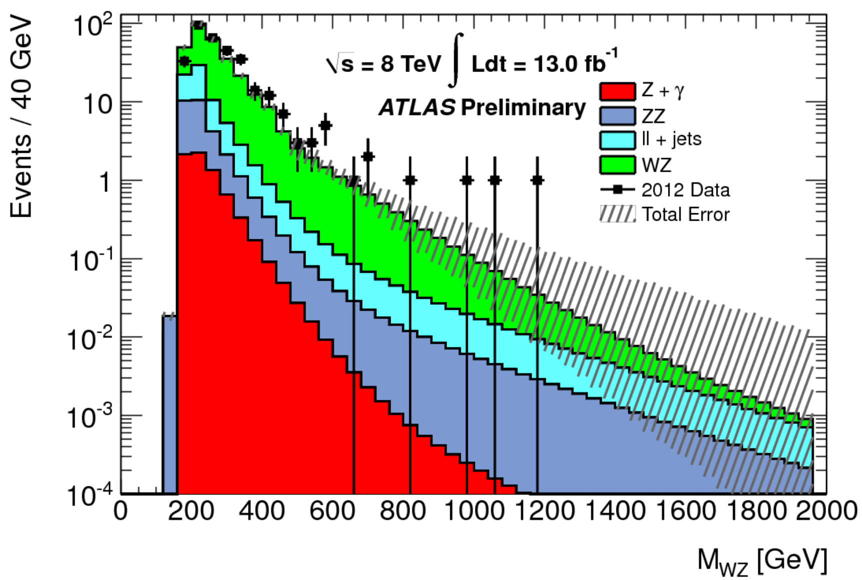


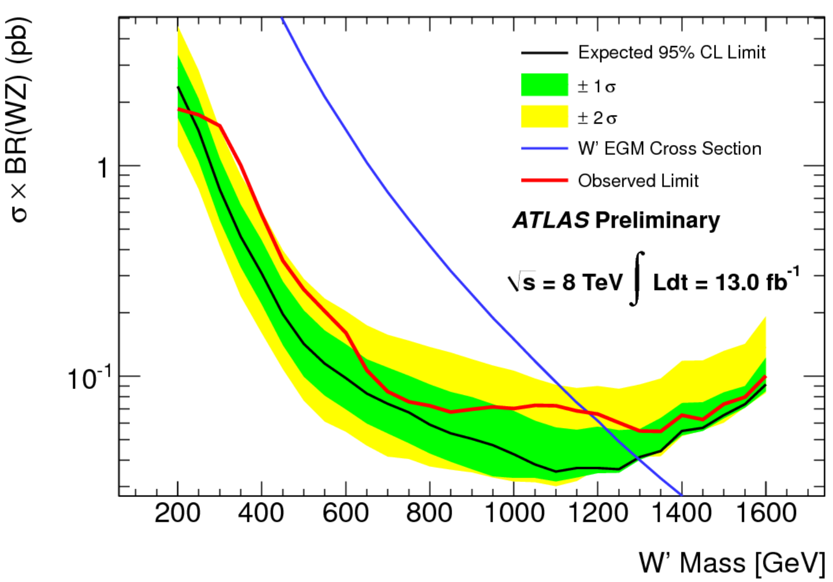


WZ resonance search

- Many theories predict di-boson resonances at high energies
- Search for ZW → lllν in final states with (ee eν), (ee μν), (μμ eν), (μμ μν)





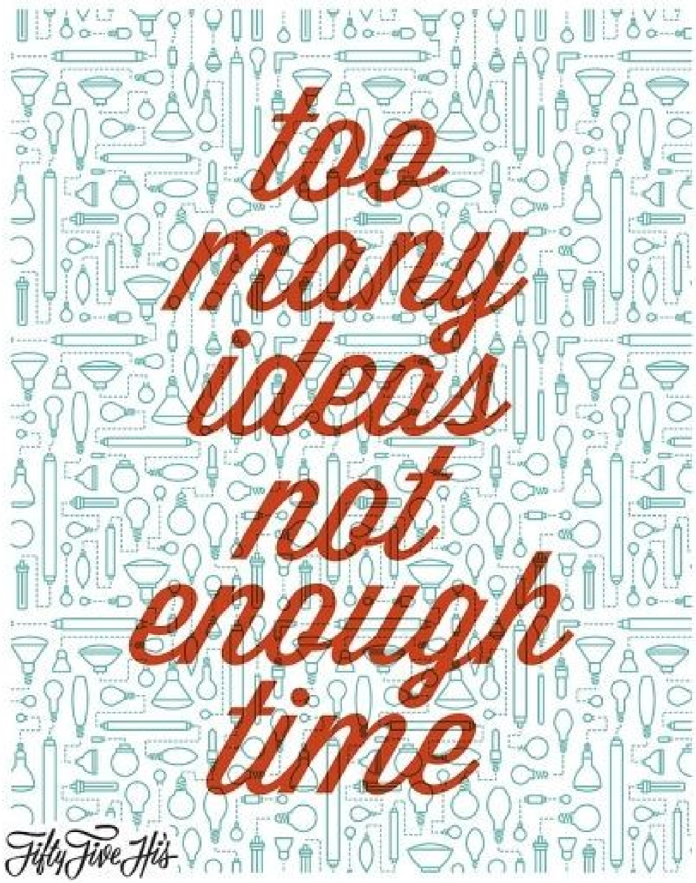


ATLAS-CONF-2013-015

Too many searches …

A lot of many interesting analyses had to be skipped, but the fun continues here:

https://twiki.cern.ch/twiki/bin/view/AtlasPublic/



Conclusions

- ATLAS supports an extensive program of searches for new phenomena in various final states
  - Neutral Higgs searches to ττ , μμ, WW; invisible decays or decays to long lived particles; Charged scalars; heavy resonances
- No evidence for new physics yet, but Nature is bound to be natural
  - New discoveries are imminent!

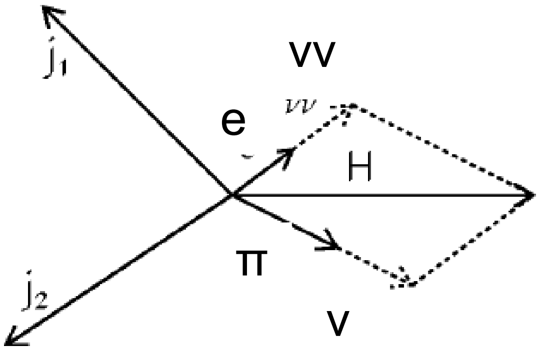
Many thanks for your attention!

Additional slides

Missing Mass Calculator

- An extension of the collinear mass approximation
  - Collinear mass: assume that neutrinos are emitted in the same direction as the visible decay products

$$\not{E}_X = P_{\nu 1} \cdot \cos(\theta_1) \cdot \cos(\varphi_1) + P_{\nu 2} \cdot \cos(\theta_2) \cdot \cos(\varphi_2)$$
$$\not{E}_Y = P_{\nu 1} \cdot \cos(\theta_1) \cdot \sin(\varphi_1) + P_{\nu 2} \cdot \cos(\theta_2) \cdot \sin(\varphi_2)$$



  - Missing mass calculator:

→ Write the full equation system: more unknowns than equations
→ parameterise the 3D angle between visible and invisible tau decay products from MC simulation, dθ
→ solve the equation on a grid of the extra unknowns and calculate the most probable choice using the dθ distribution

$$E_x^{miss} = p_{\text{mis}_1} \sin\theta_{\text{mis}_1} \cos\phi_{\text{mis}_1} + p_{\text{mis}_2} \sin\theta_{\text{mis}_2} \cos\phi_{\text{mis}_2}$$
$$E_y^{miss} = p_{\text{mis}_1} \sin\theta_{\text{mis}_1} \sin\phi_{\text{mis}_1} + p_{\text{mis}_2} \sin\theta_{\text{mis}_2} \sin\phi_{\text{mis}_2}$$
$$M_{\tau_1}^2 = m_{\text{mis}_1}^2 + m_{\text{vis}_1}^2 + 2\sqrt{p_{\text{vis}_1}^2 + m_{\text{vis}_1}^2}\sqrt{p_{\text{mis}_1}^2 + m_{\text{mis}_1}^2} - 2 p_{\text{vis}_1} p_{\text{mis}_1} \cos\Delta\theta_{\nu m_1}$$
$$M_{\tau_2}^2 = m_{\text{vis}_2}^2 + 2\sqrt{p_{\text{vis}_2}^2 + m_{\text{vis}_2}^2}\sqrt{p_{\text{mis}_2}^2 + m_{\text{mis}_2}^2} - 2 p_{\text{vis}_2} p_{\text{mis}_2} \cos\Delta\theta_{\nu m_2}$$

2HDM Parameters

CP-conserving 2HDM with softly broken Z_2 symmetry

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \left(\Phi_2^\dagger \Phi_1\right)^2\right],$$

$$\langle \Phi_1 \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}, \qquad \langle \Phi_2 \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix}$$

Makes 8 parameters. Consider already known u we have 7 left.
We can write them as 4 masses of the (pseudo-)scalars (h,H,A,$H^\pm$), tan β = u2/u1, α (=mixing angle between h and H) and m_{12}.

	Type I	Type II	Lepton-specific	Flipped
ξ_h^u	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$
ξ_h^d	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$
ξ_h^ℓ	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$
ξ_H^u	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$
ξ_H^d	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$
ξ_H^ℓ	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos\alpha/\cos\beta$	$\sin\alpha/\sin\beta$
ξ_A^u	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
ξ_A^d	$-\cot\beta$	$\tan\beta$	$-\cot\beta$	$\tan\beta$
ξ_A^ℓ	$-\cot\beta$	$\tan\beta$	$\tan\beta$	$-\cot\beta$